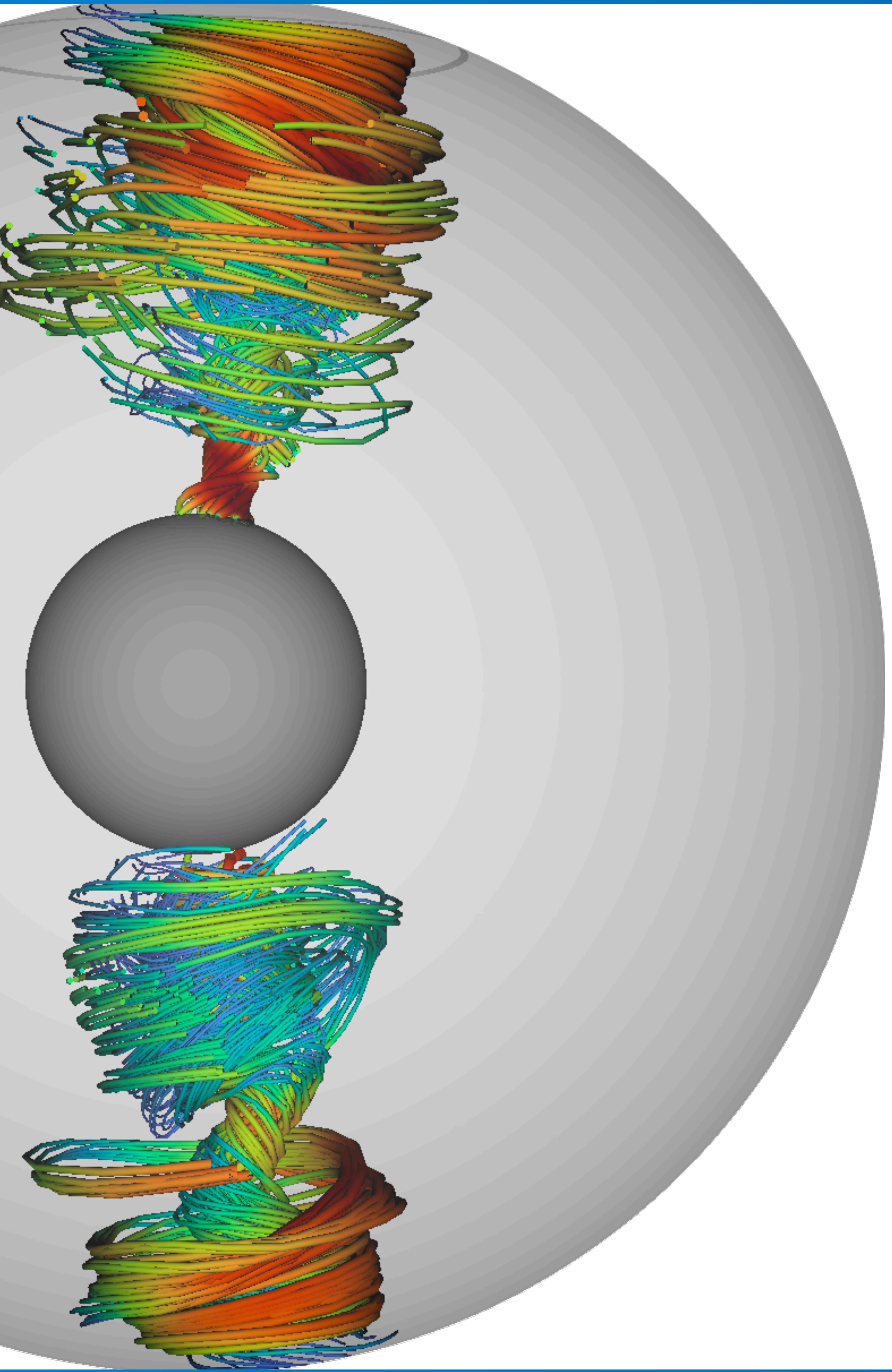

New branch of the Tayler instability-driven dynamo in 3D numerical simulations of stellar radiative zones

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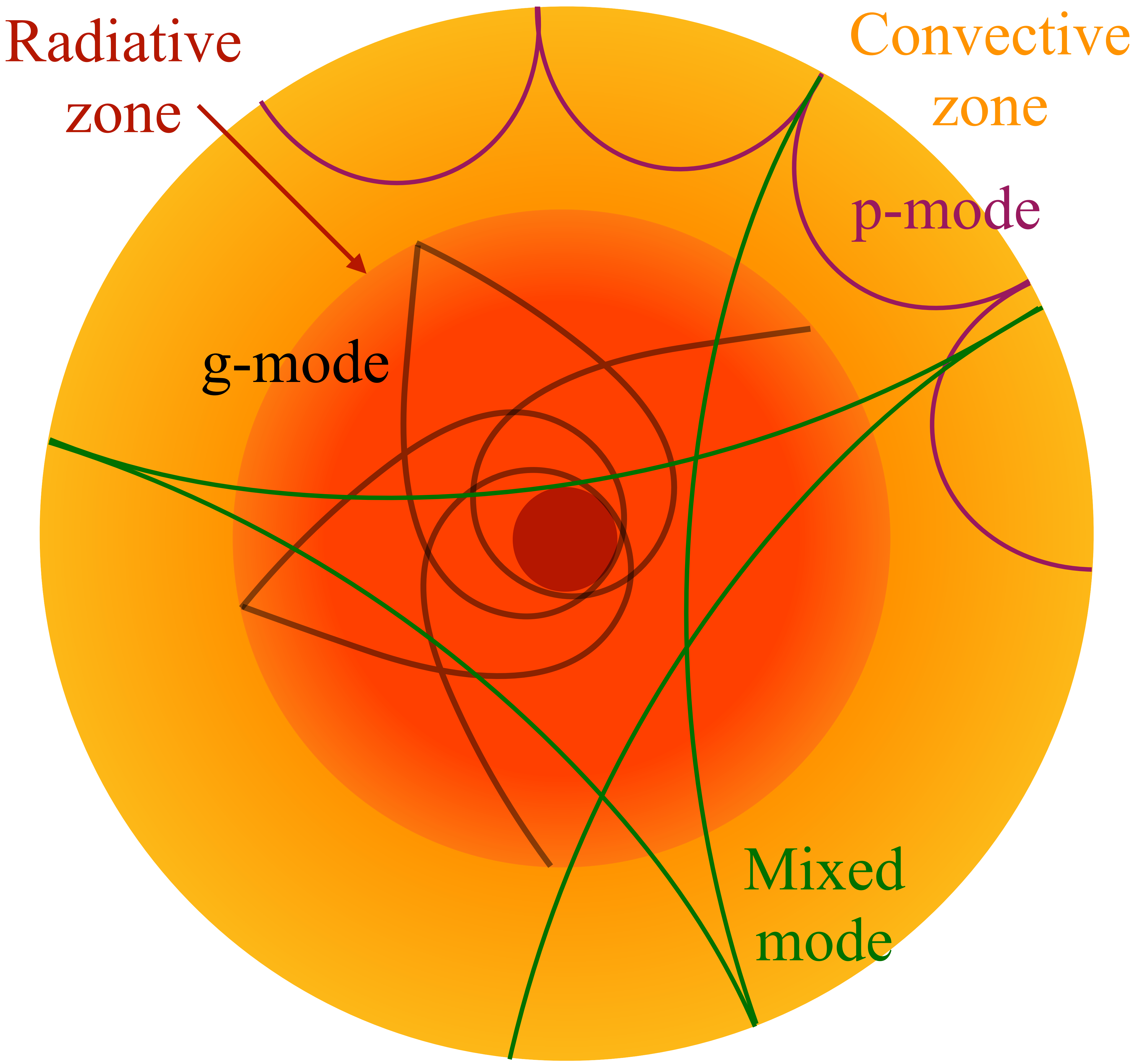
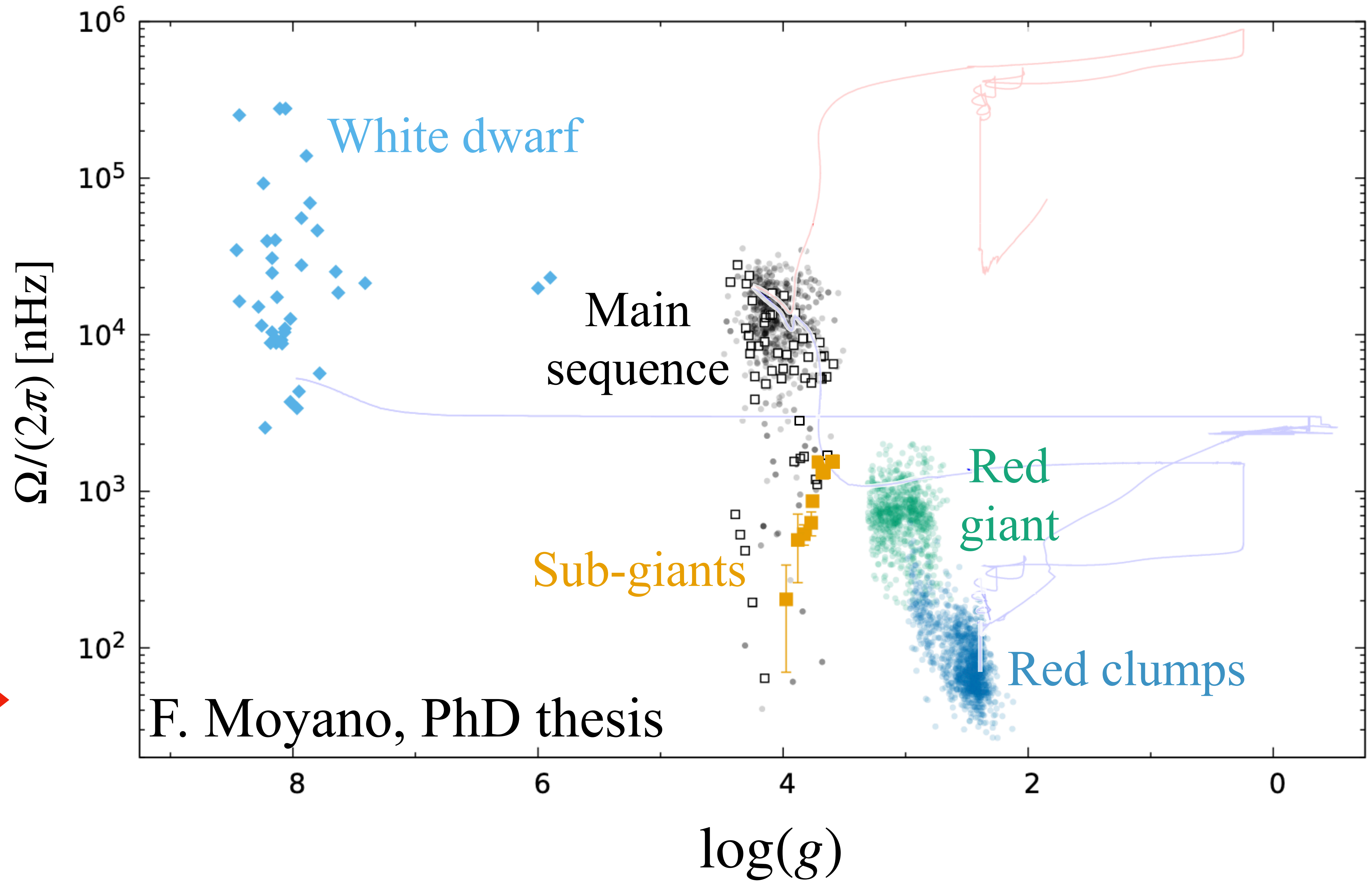
17th December 2025

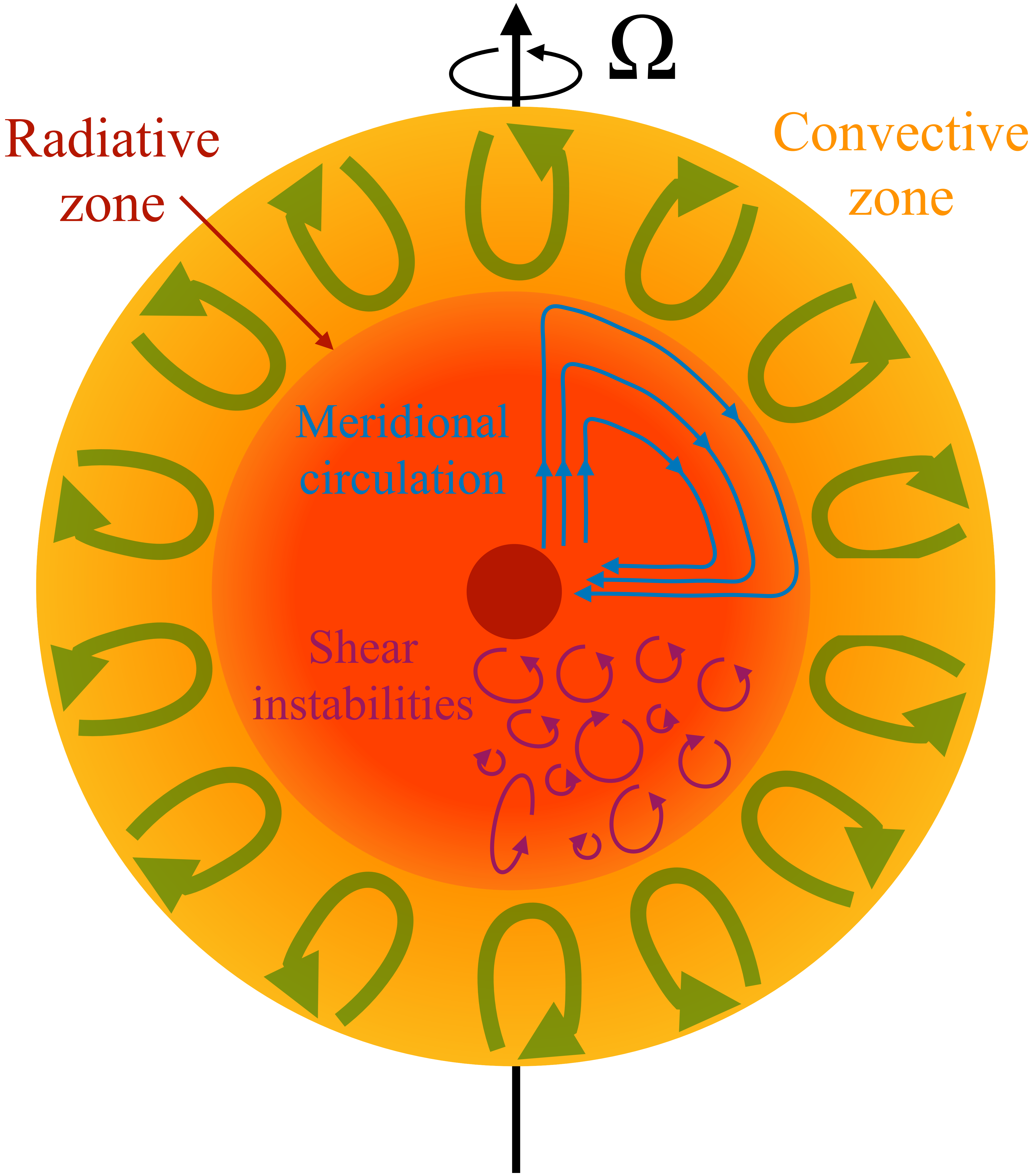
UNIVERSITÉ DE GENÈVE

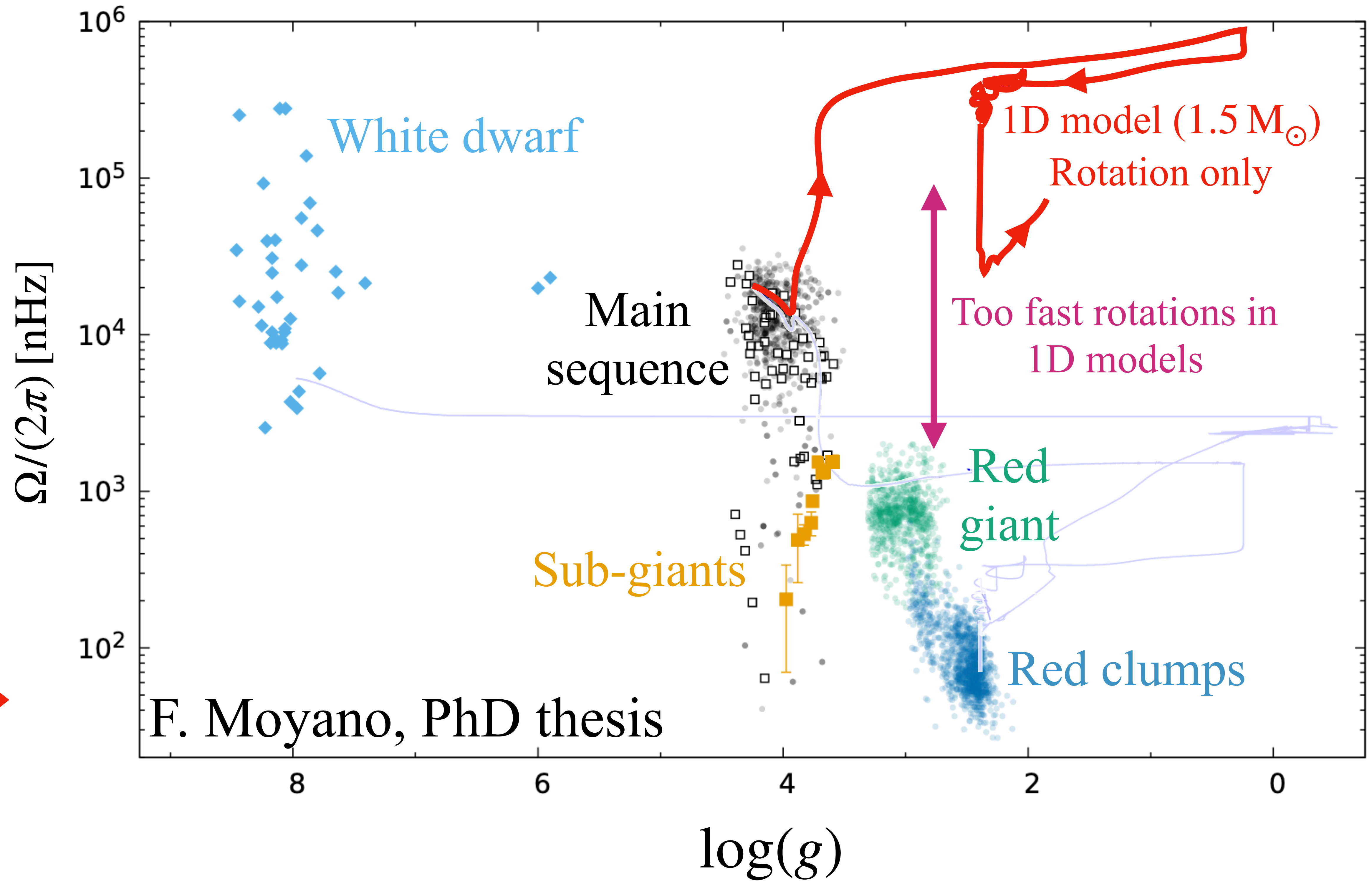
Rotation of stellar interior through asteroseismology





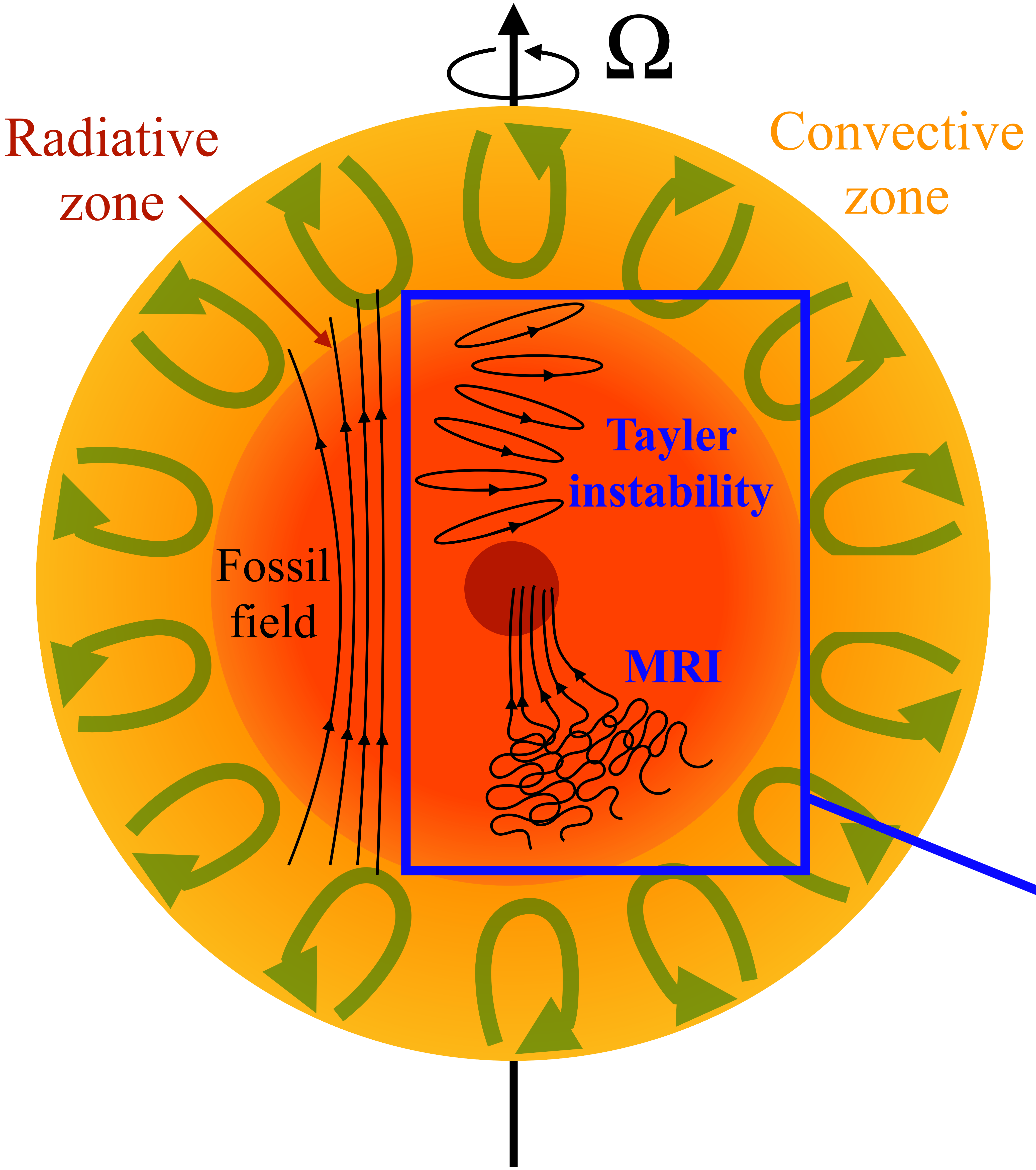
Descrepancy with 1D rotating stellar evolution models

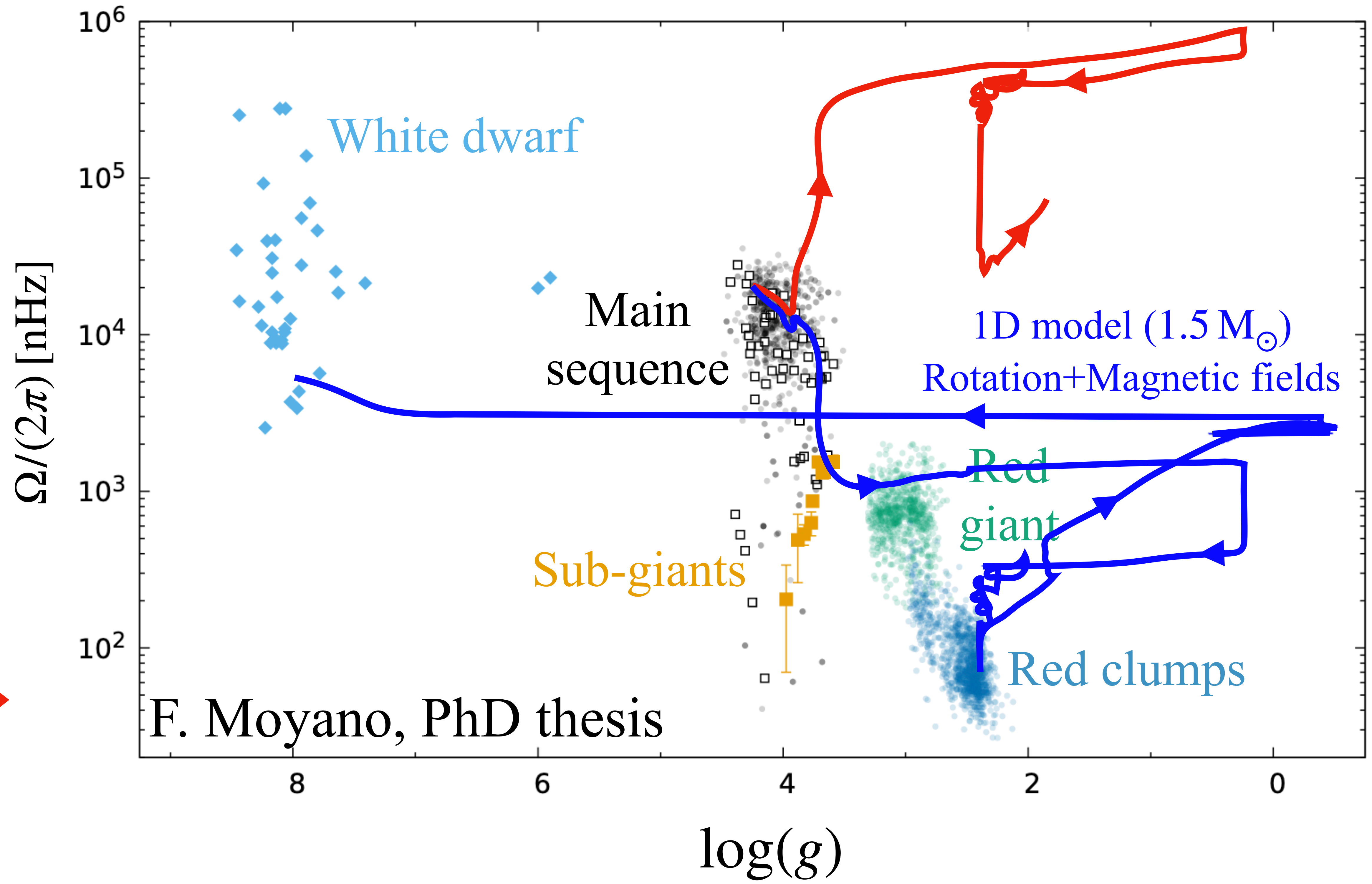




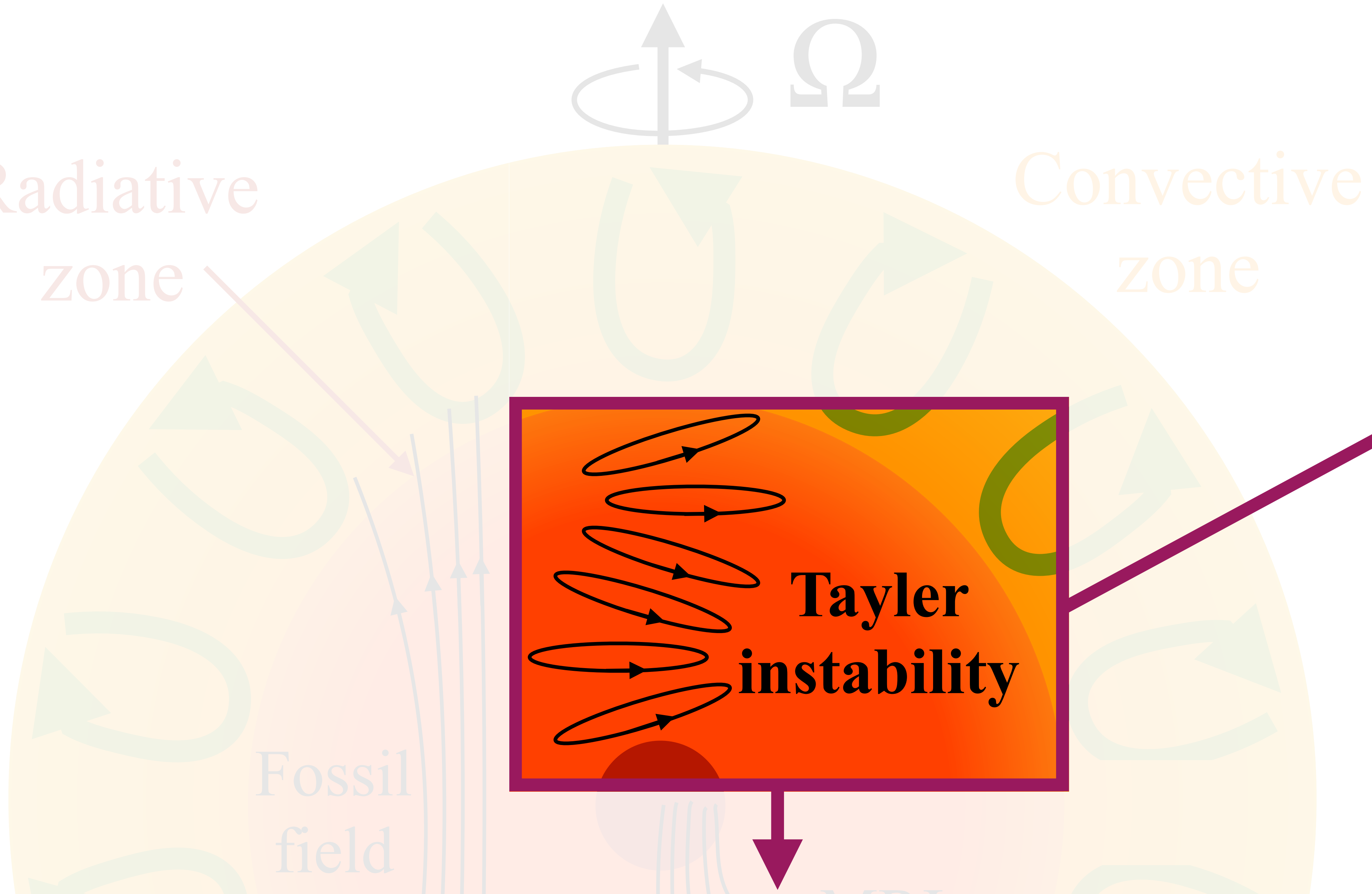
$\Longrightarrow$ Other efficient AM transport mechanisms needed

Transport by magnetic fields





Large-scale magnetic fields produced by
subcritical dynamos
driven by MHD instabilities

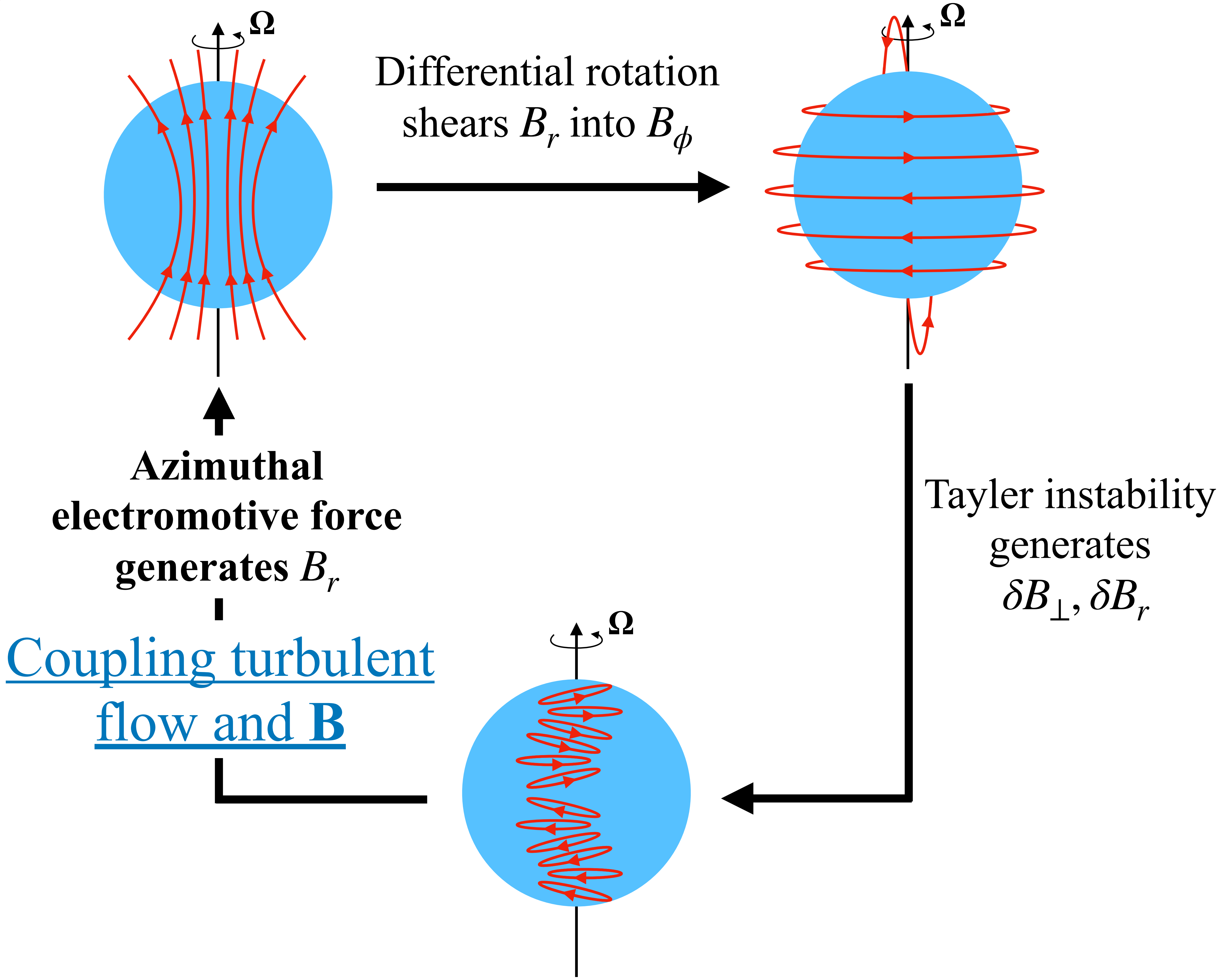


Analytical studies

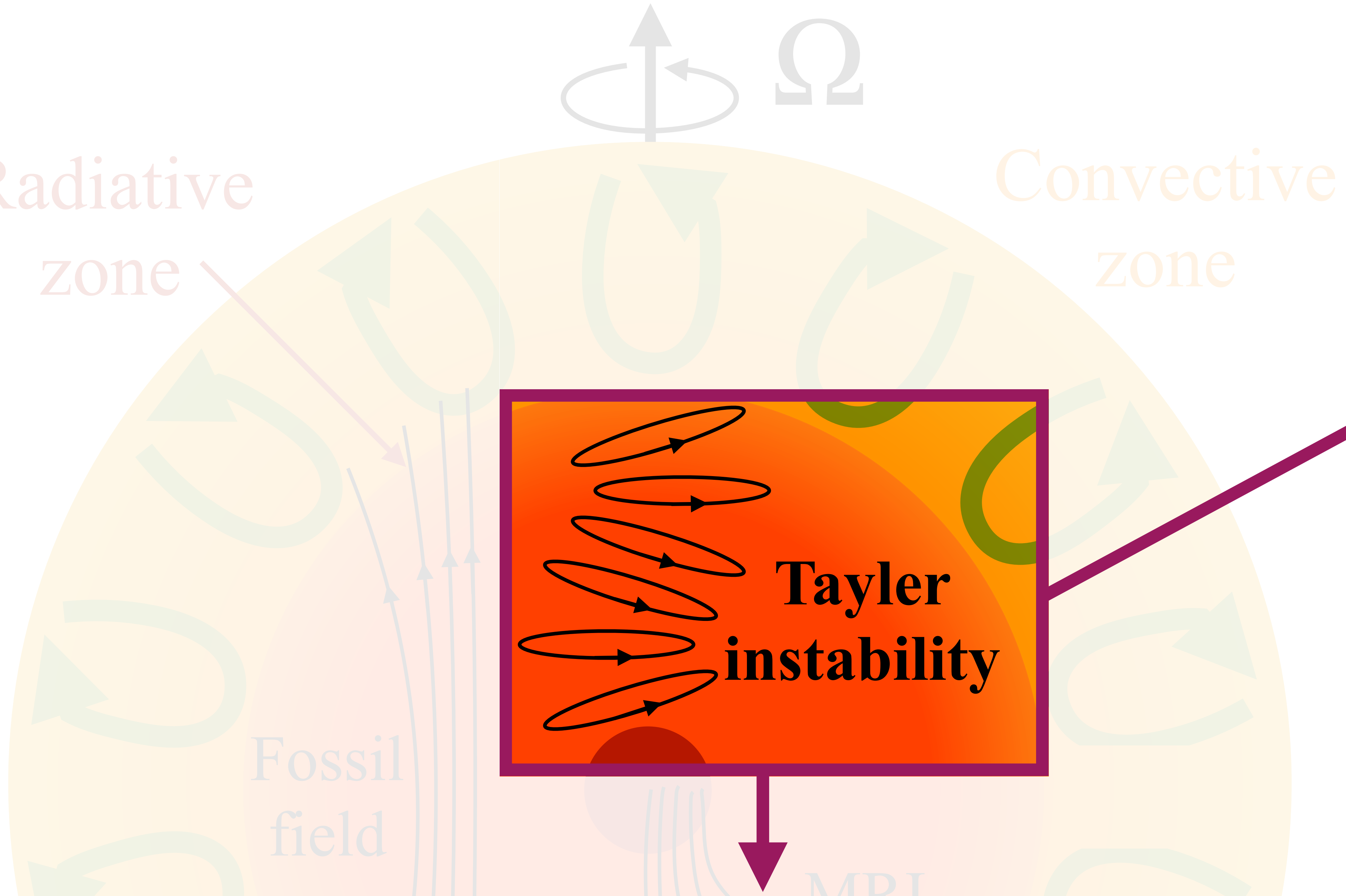
Spruit 2002	Fuller+2019
$B_\phi/\sqrt{4\pi\rho r^2} \sim q\Omega^2/N$	$B_\phi/\sqrt{4\pi\rho r^2} \sim q^{1/3}\Omega^{4/3}/N^{1/3}$
$B_r/\sqrt{4\pi\rho r^2} \sim q^2\Omega^4/N^3$	$B_r/\sqrt{4\pi\rho r^2} \sim q^{2/3}\Omega^{8/3}/N^{5/3}$

- Easy to implement in 1D evolution models
- Strong assumptions on nonlinear induction and dissipation
- No information on the 3D field geometry

Tayler instability-driven dynamo
(Tayler-Spruit dynamo)

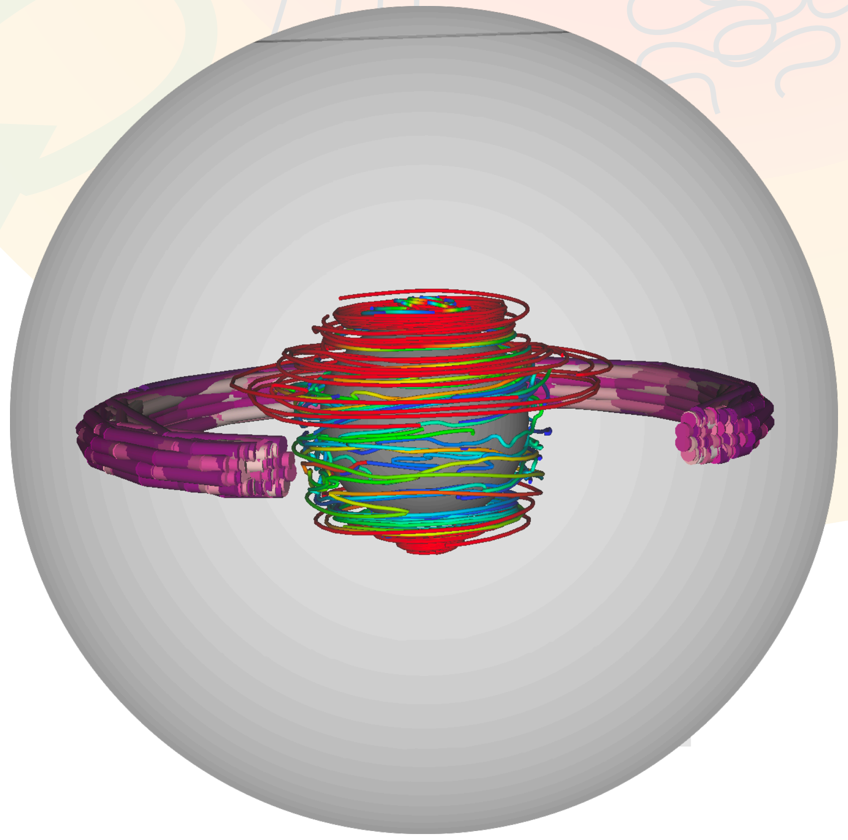


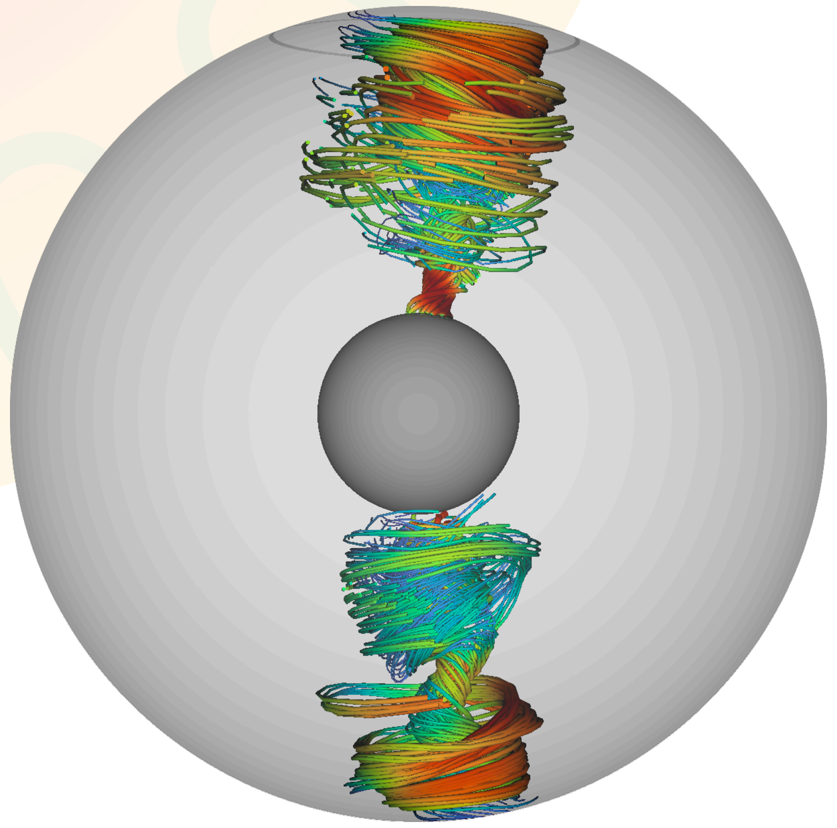
Necessity of 3D models



3D direct numerical simulations

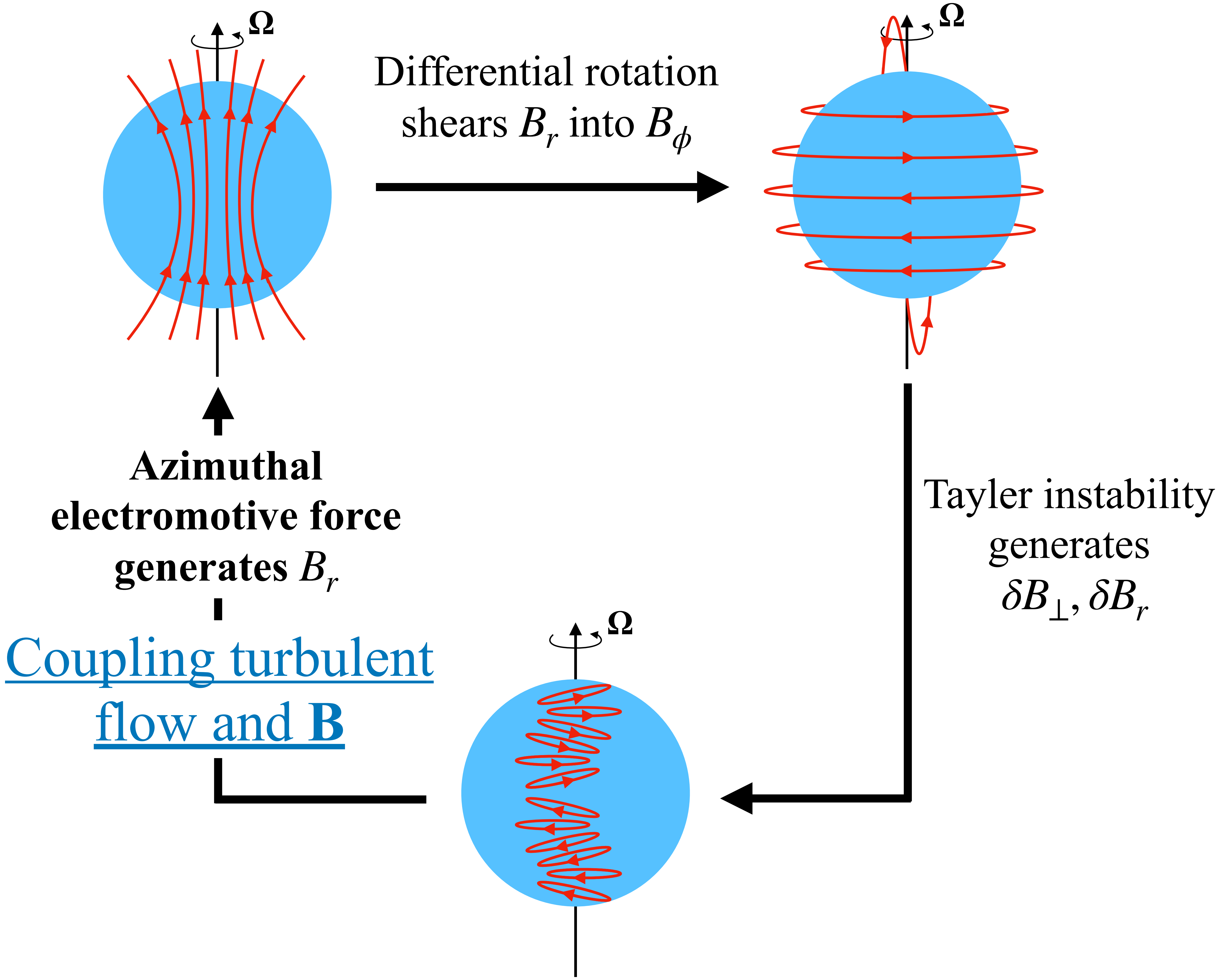
(Petitdemange+23,24) (Barrère+23,25,subm.)





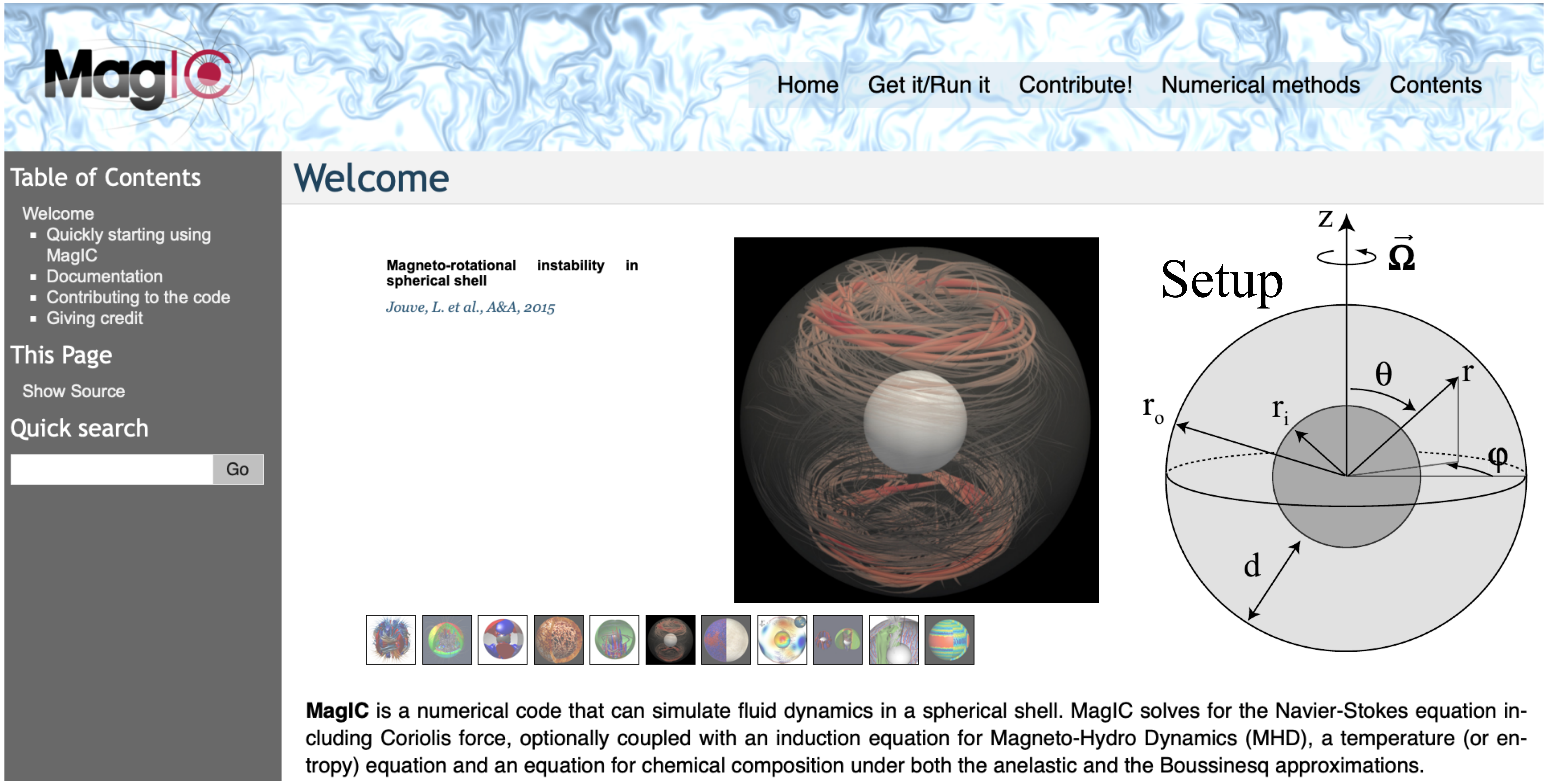
Tayler instability-driven dynamo

(Tayler-Spruit dynamo)



Open source code: MagIC

- **Pseudo-spectral code**
  - **r**: Chebyshev polynomial
  - θ and ϕ: Sphericals harmonics
- **Parallelisation**
  - MPI
  - OpenMP
- **Open source !**
  - magic-sph.github.io



(Wicht 2002, Gastine & Wicht 2012, Schaeffer 2013)

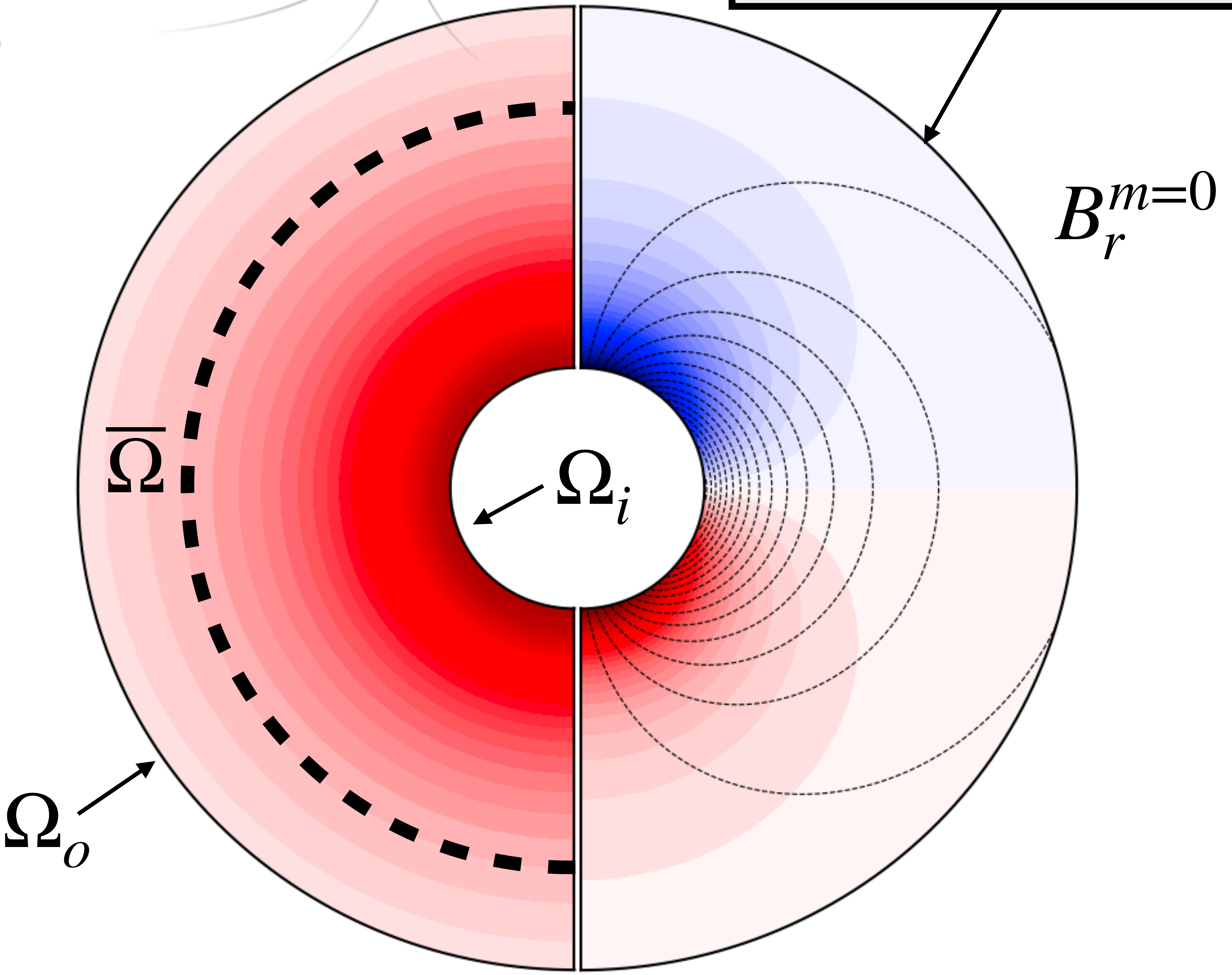
Boussinesq approximation:

- **Uniform** density profile
- Density perturbations **ONLY** in buoyancy term

- Less costly in CPU hours (~**100 kCPU hours per DNS**)
  $\Longrightarrow$ Easier to produce parametric studies
- Unrealistic in stars

Initial magnetic field

- weak or strong
- Axial dipole or quadrupole

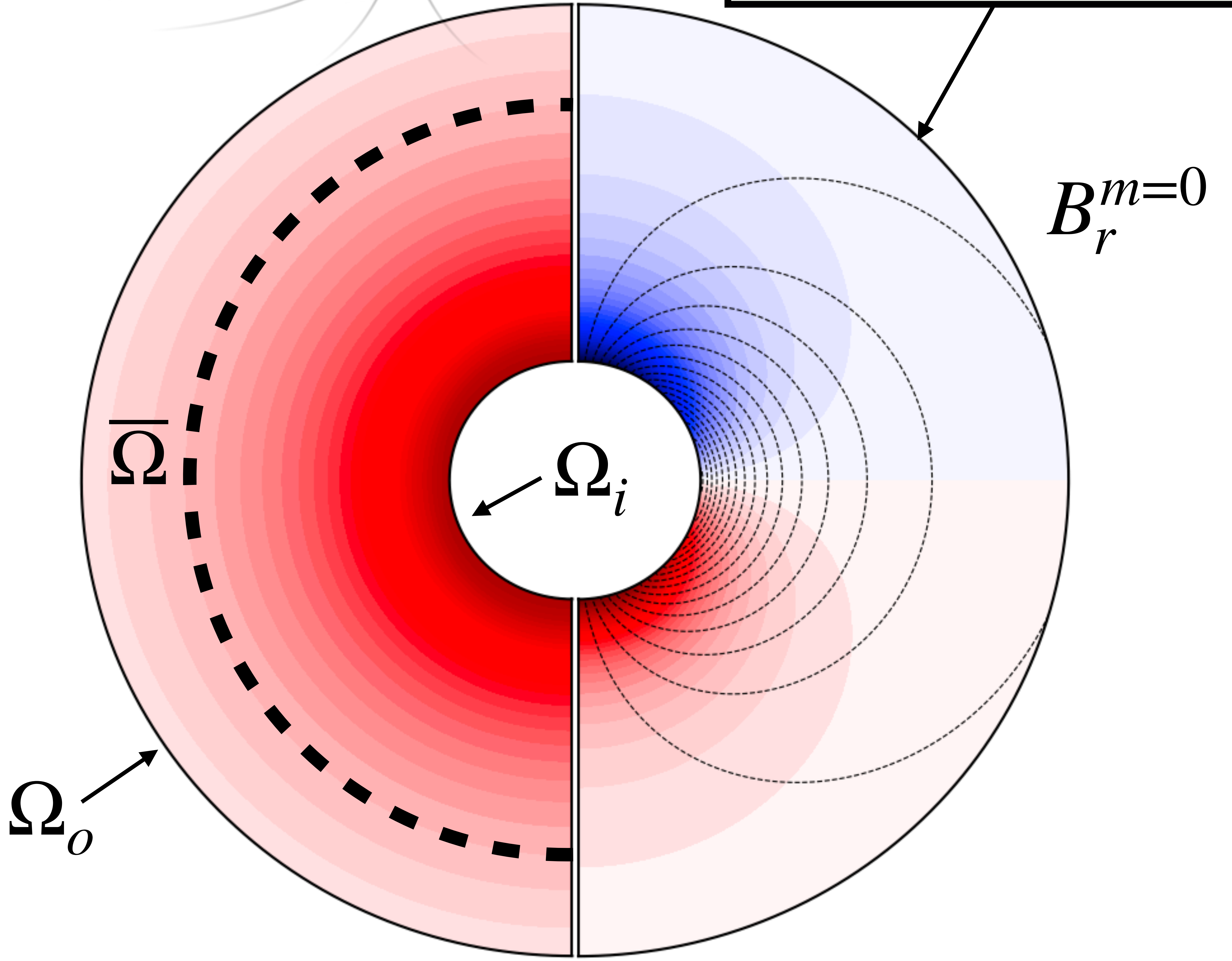


Numerical setup

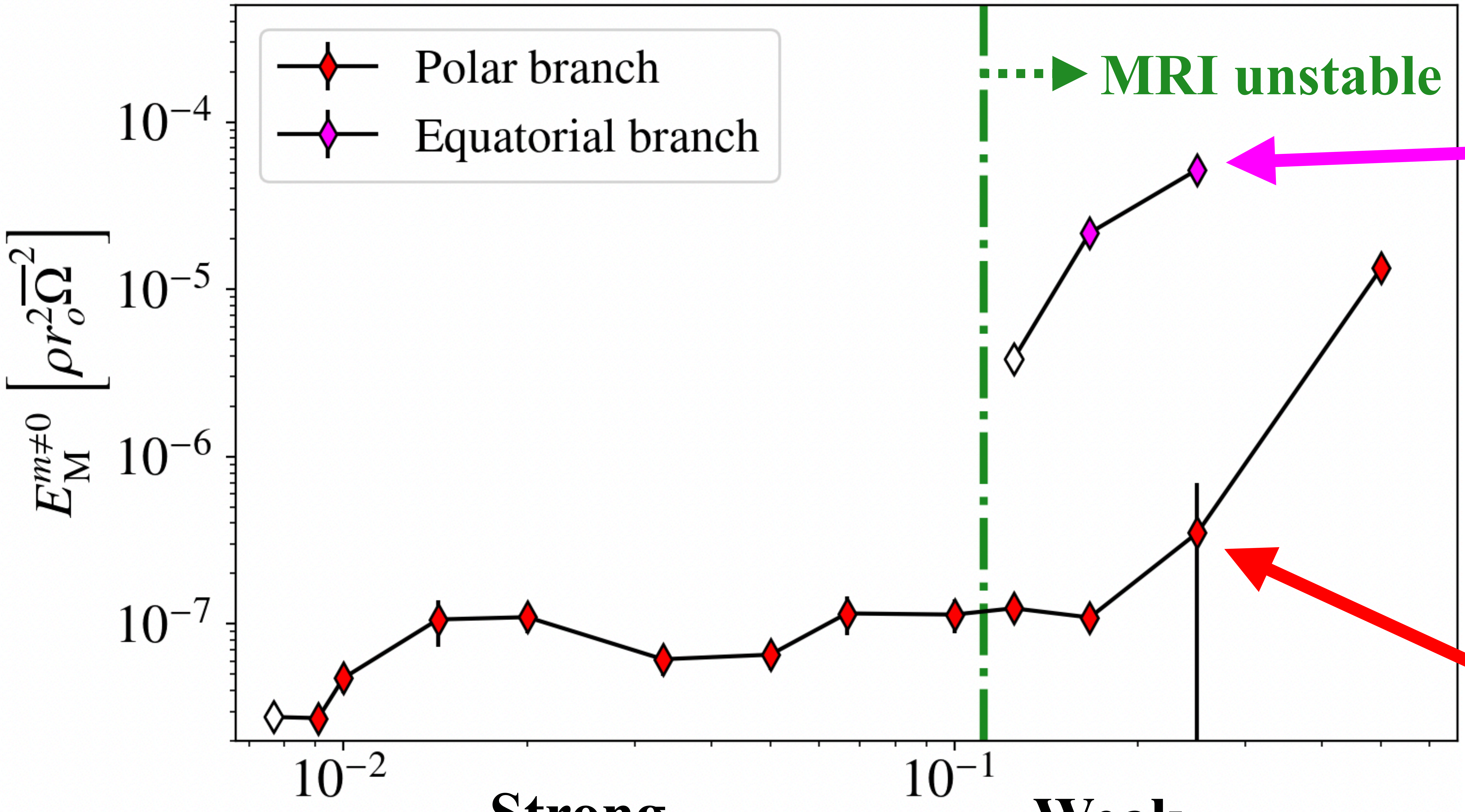
<u>Boussinesq approximation:</u>
- **Uniform** density profile
- Density perturbations **ONLY** in buoyancy term

- **Explicit diffusivities**
  $Pr = \nu/\kappa = 0.1 \gg 10^{-5}$
  $Pm = \nu/\eta = 4$
- **Stably stratified**
  $N/\overline{\Omega} \in [2, 130]$
- **Volumetric forcing of differential rotation**
  Negative shear $\Omega_i > \Omega_o$
  Shear rate $q \equiv \dfrac{r}{\Omega}\dfrac{\partial\Omega}{\partial r}$

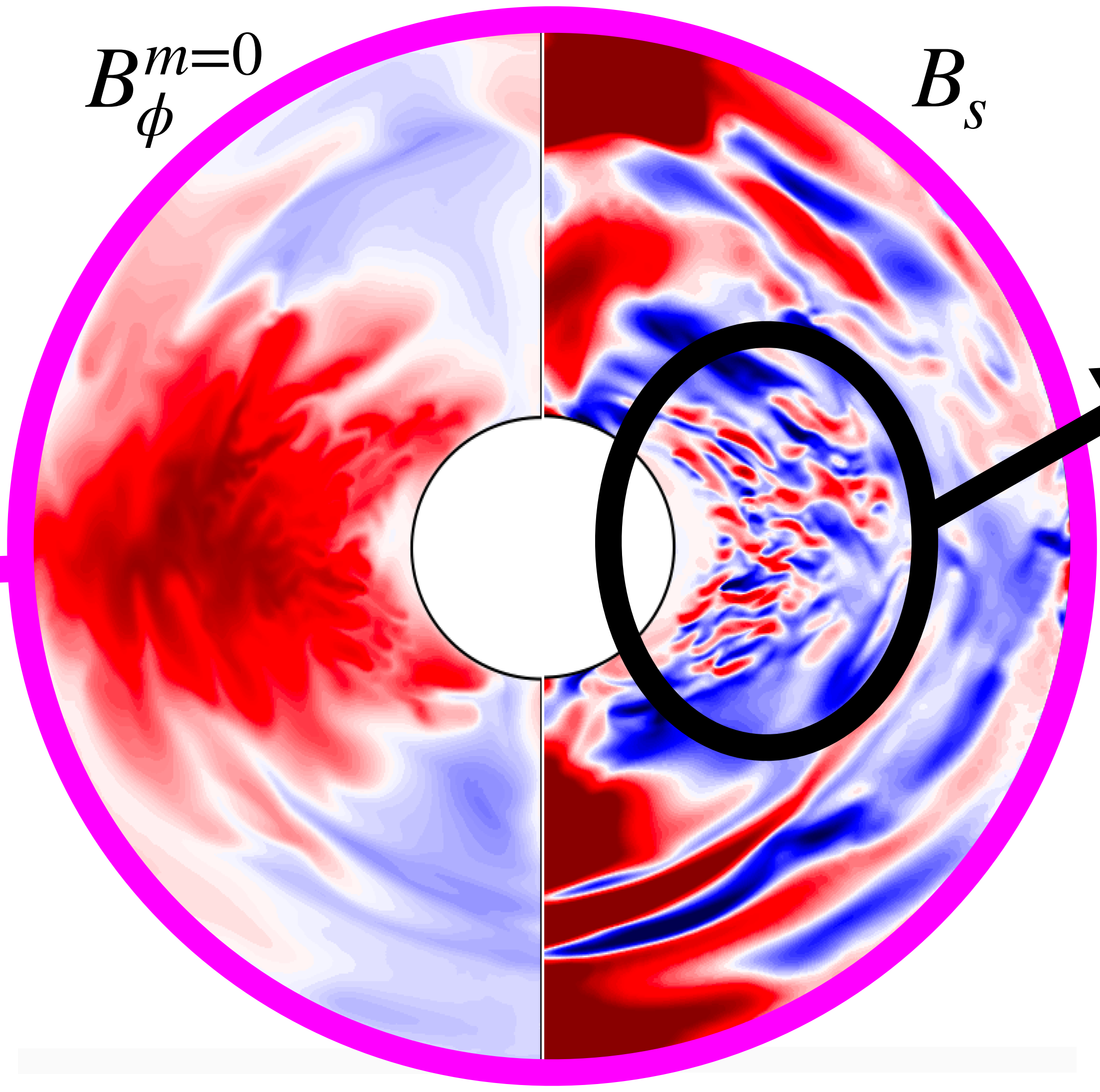
- Medium resolution: (257, 256, 512)
- High resolution: (384,384,768)



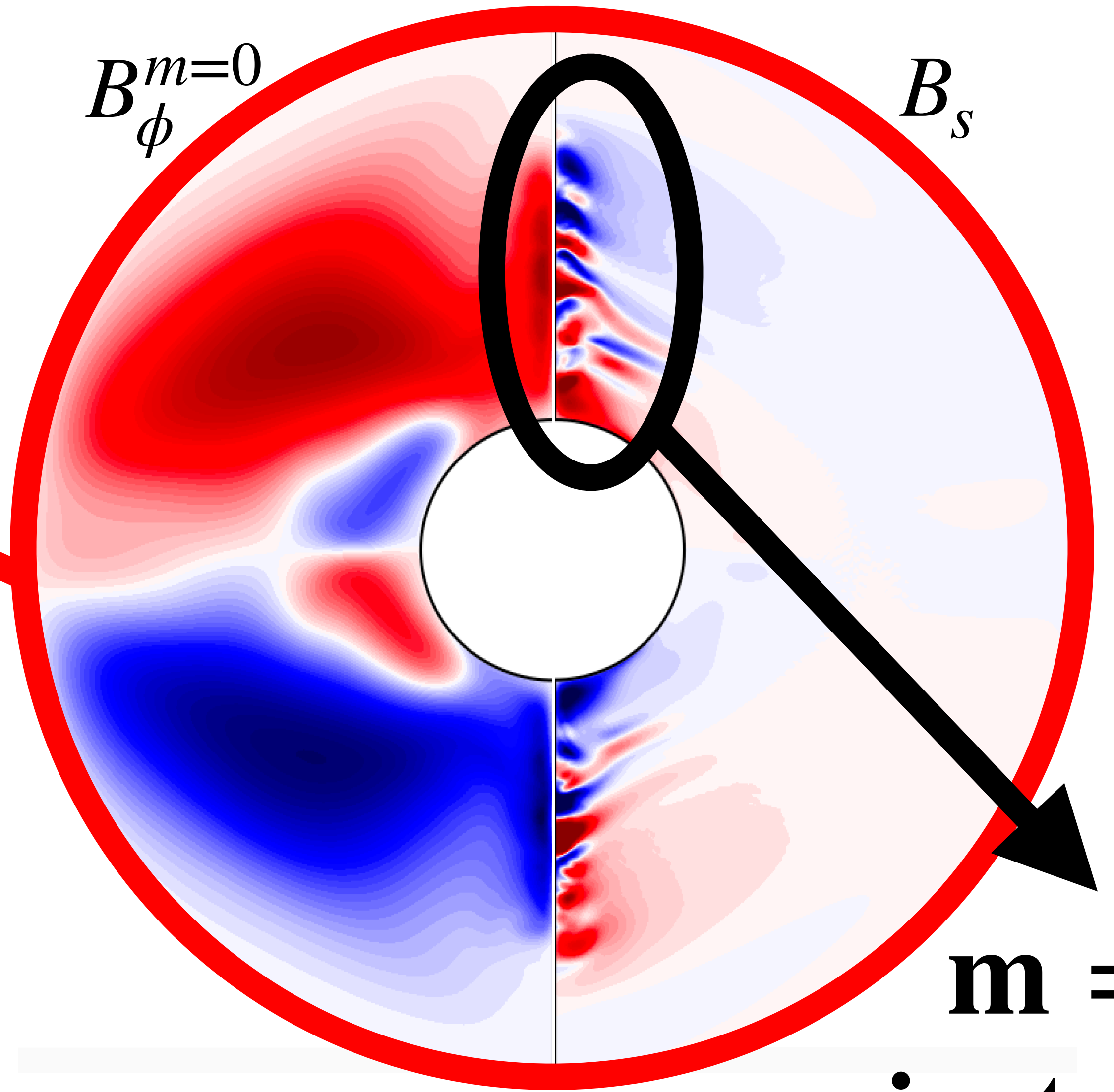
Different regimes of the dynamo



Strong stratification ← → **Weak stratification**

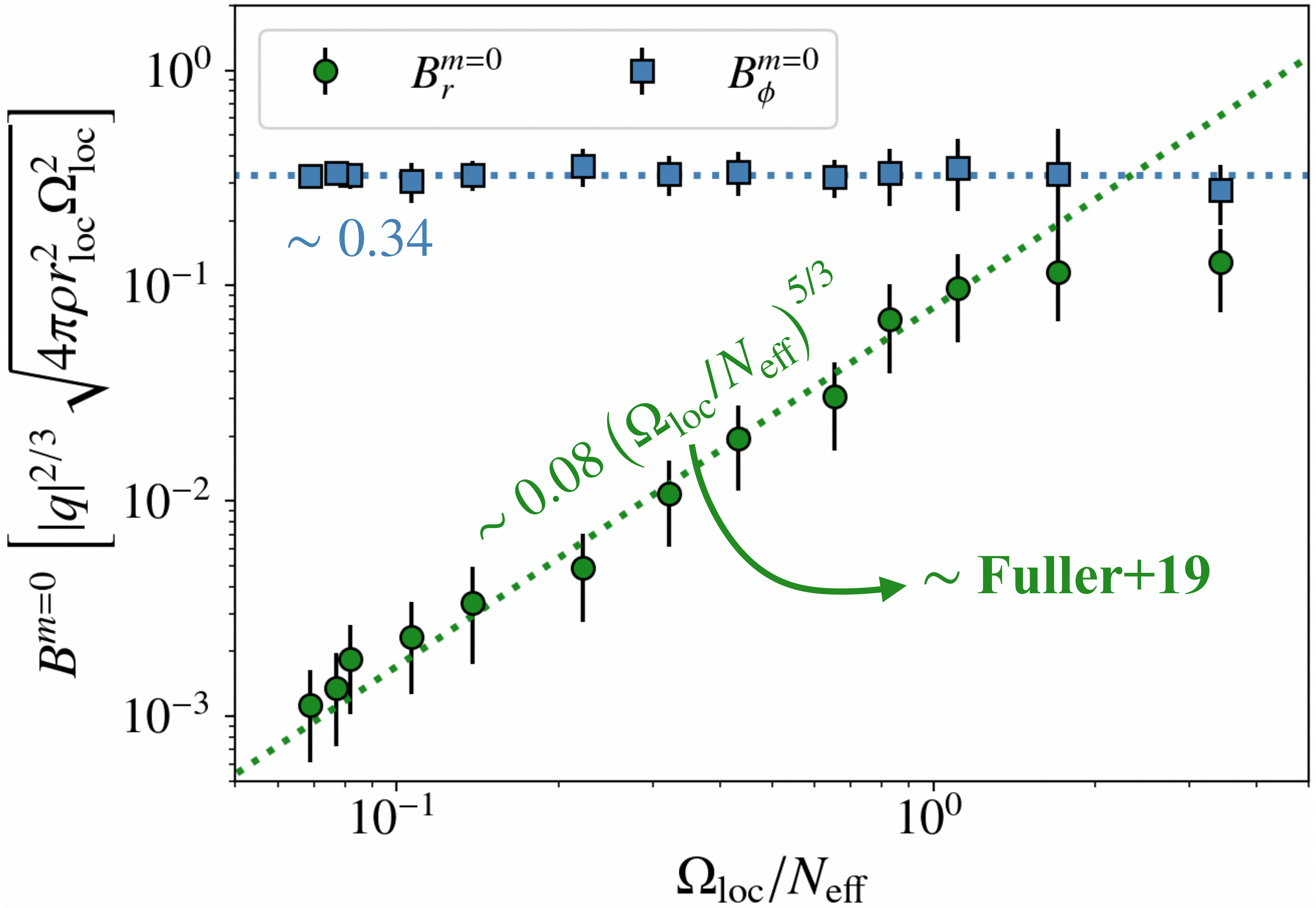


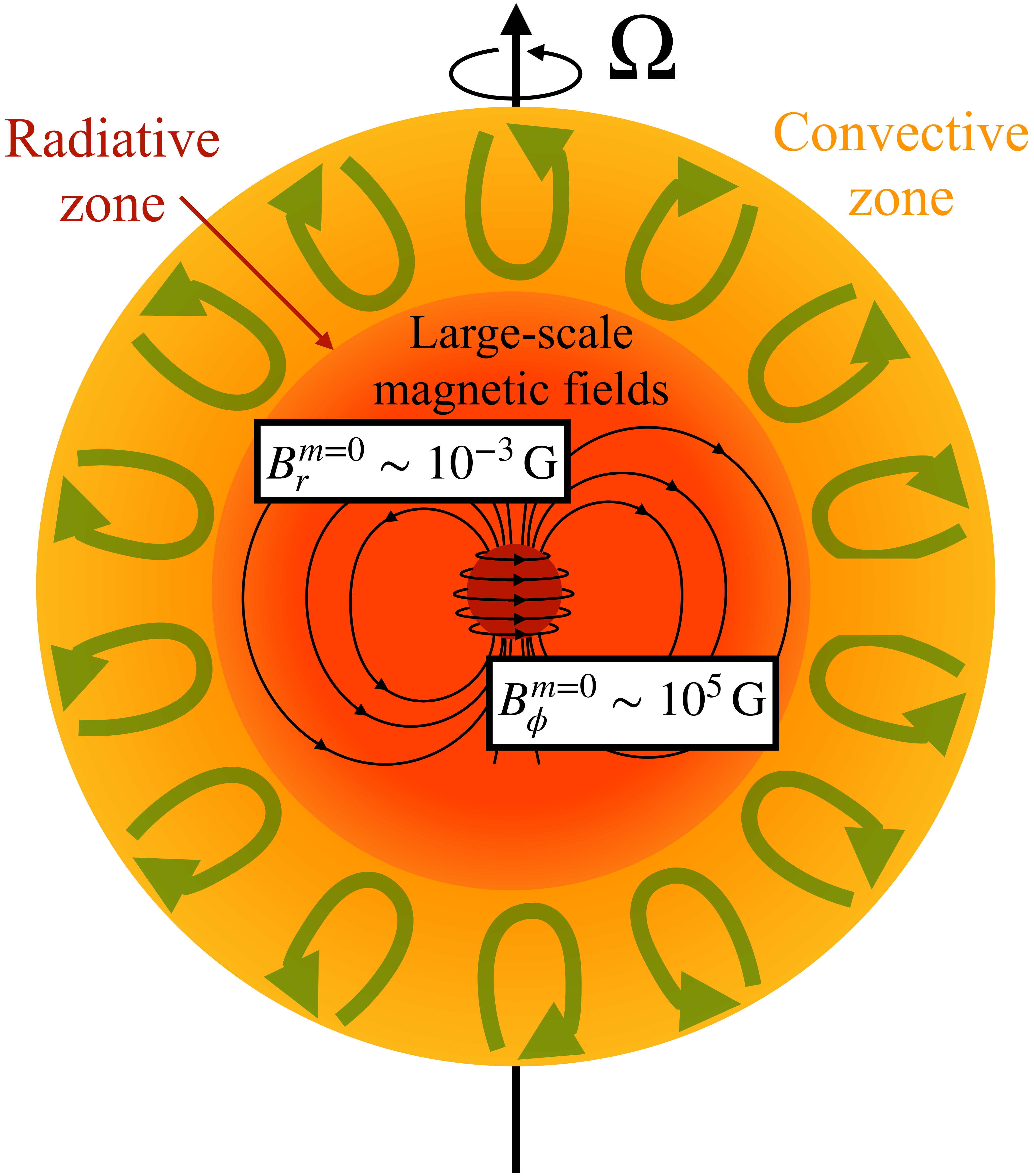
Modes only appear for $q \neq 0$ $\Longrightarrow$ **MRI ?**



m = 1 – Tayler instability modes

Large-scale magnetic fields



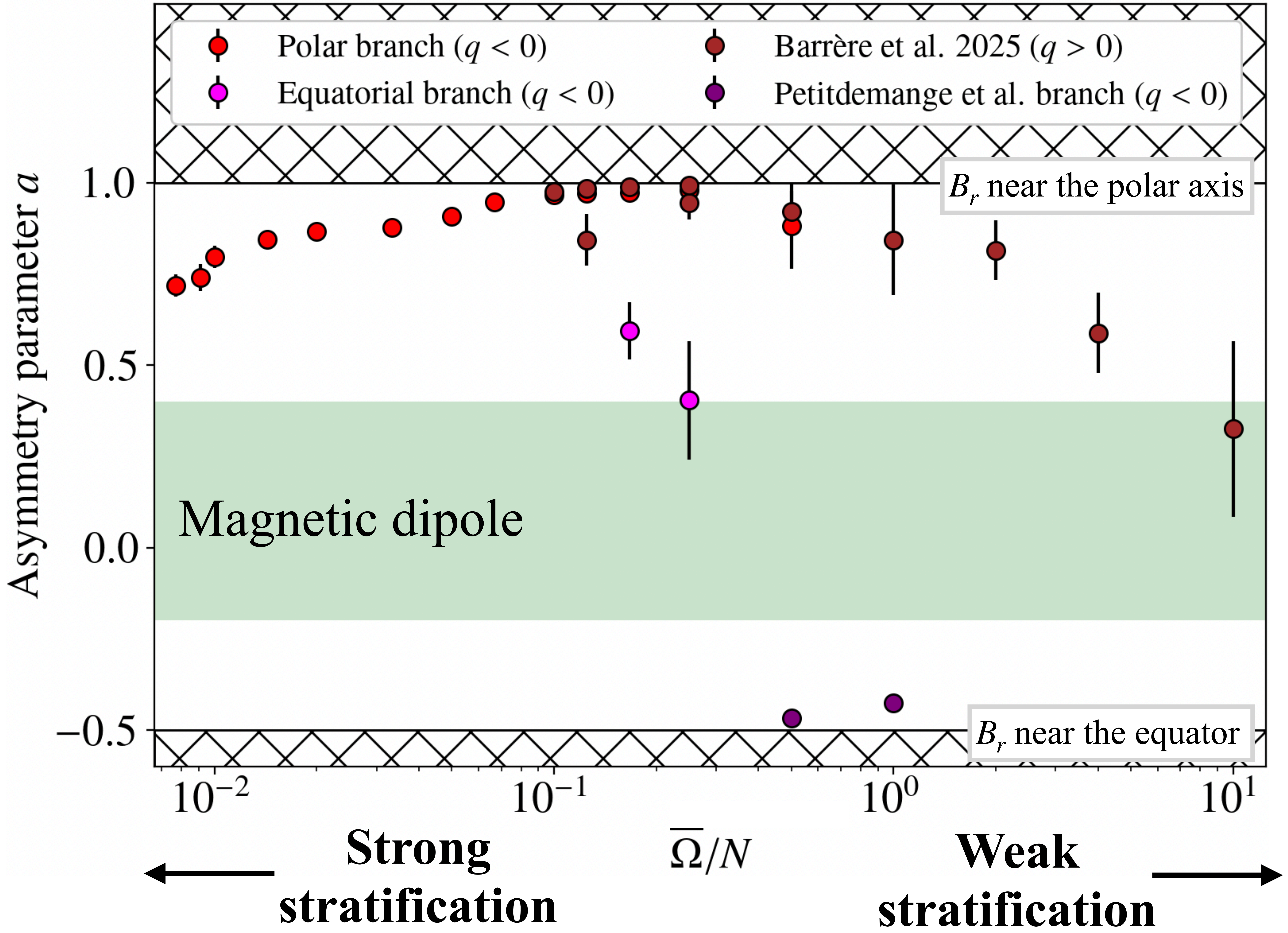


Radial magnetic field topology

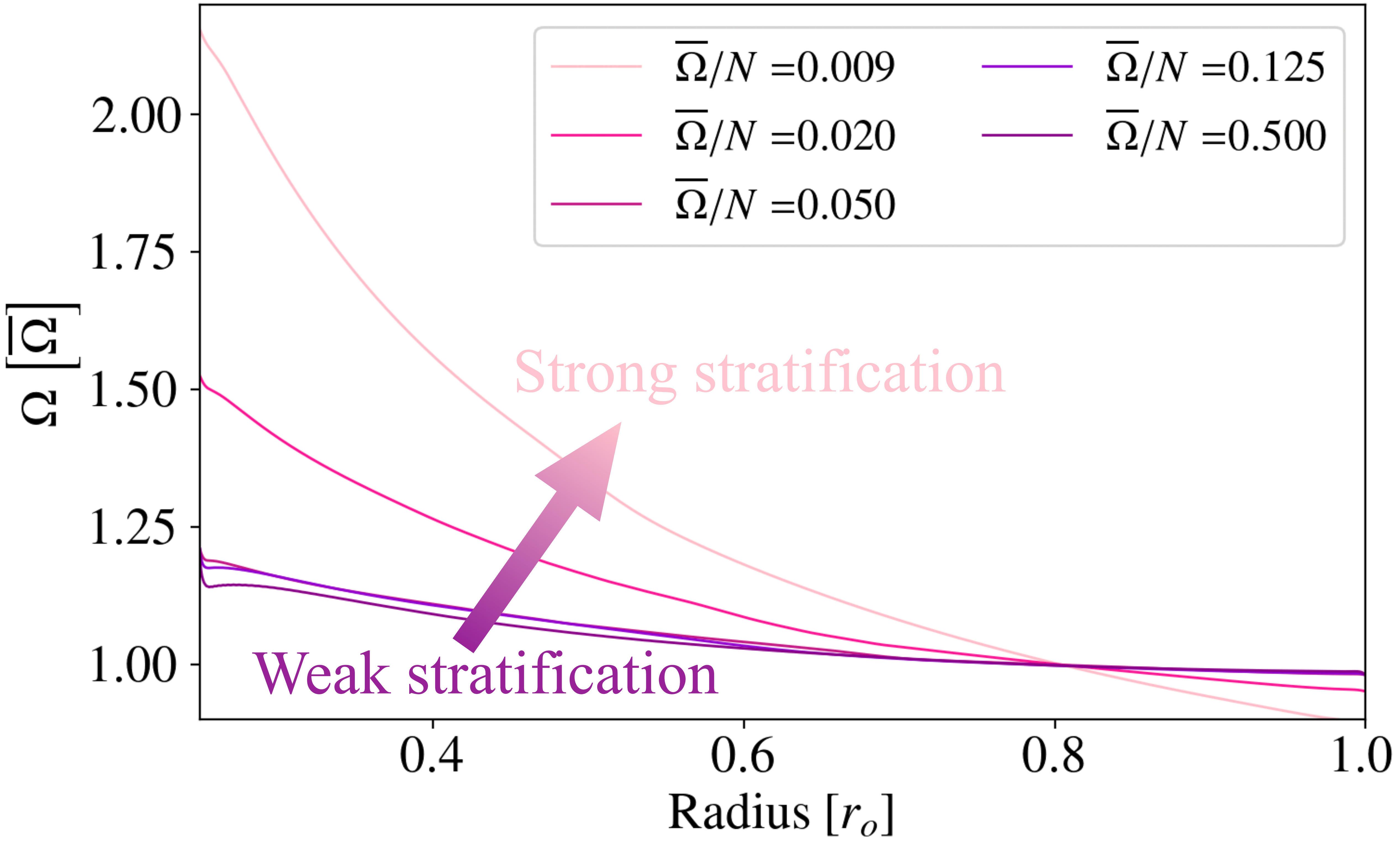
Asymmetry parameter

$$a \equiv \frac{\int_{r_i}^{r_o} K(r) \iint_S B_r^2 P_2(\cos\theta) d\Omega dr}{\int_{r_i}^{r_o} K(r) \iint_S B_r^2 d\Omega dr}$$

Weight function probing $g-$mode cavity

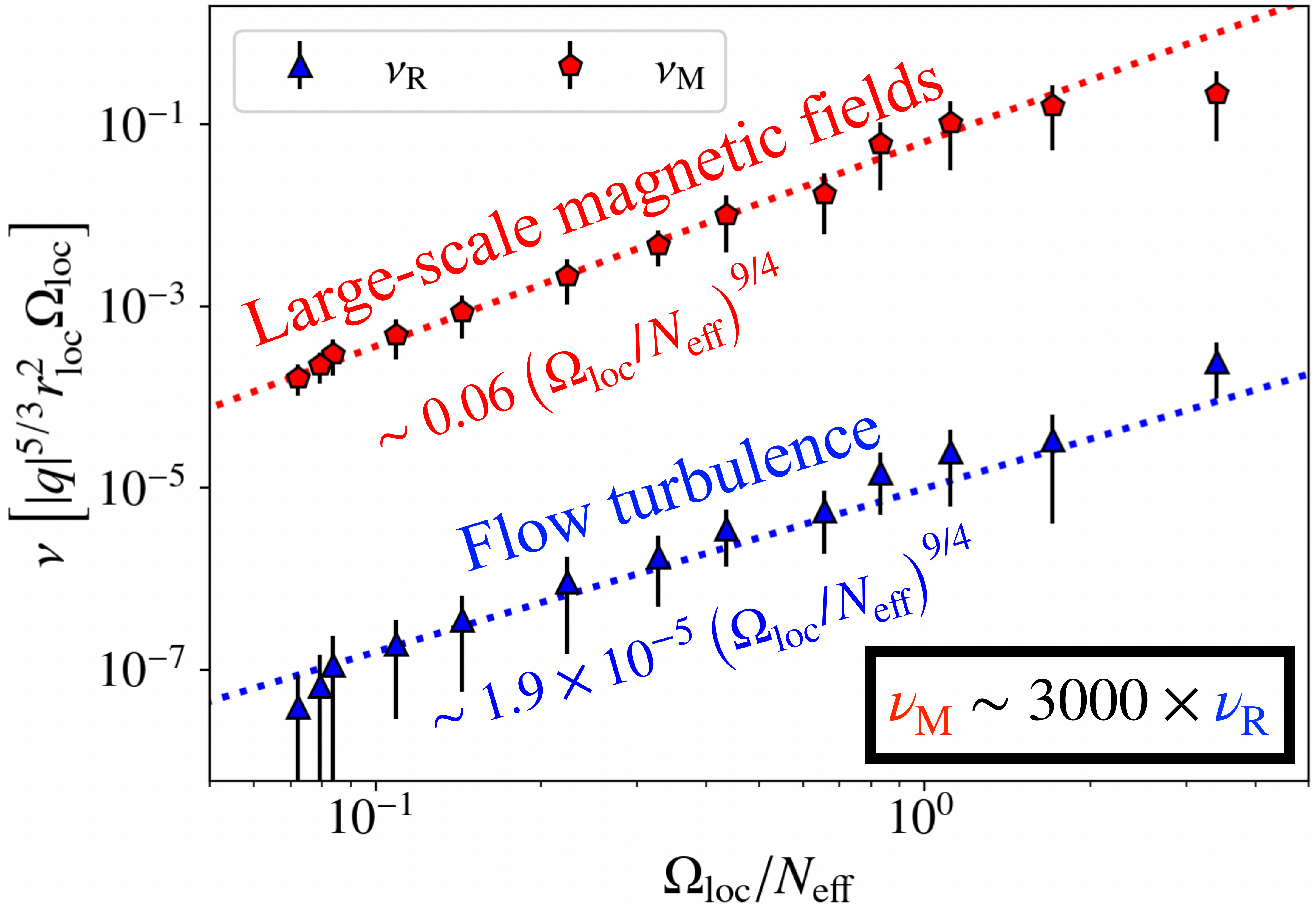


Angular momentum transport



Fuller+19:

$\nu_{\rm M} \sim 0.06\, q r_{\rm loc}^2 \Omega_{\rm loc} \left(\Omega_{\rm loc}/N_{\rm eff}\right)^2$

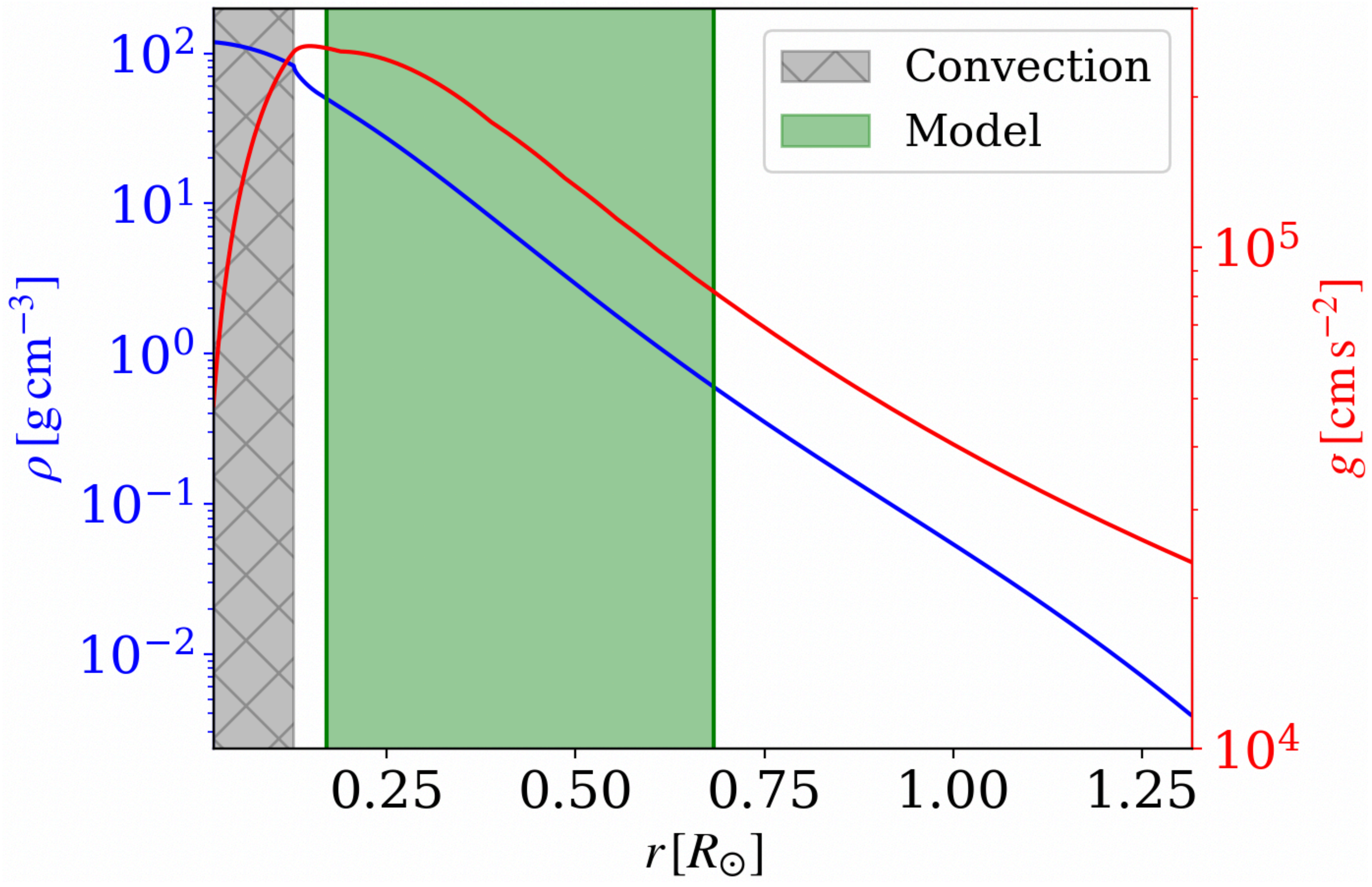


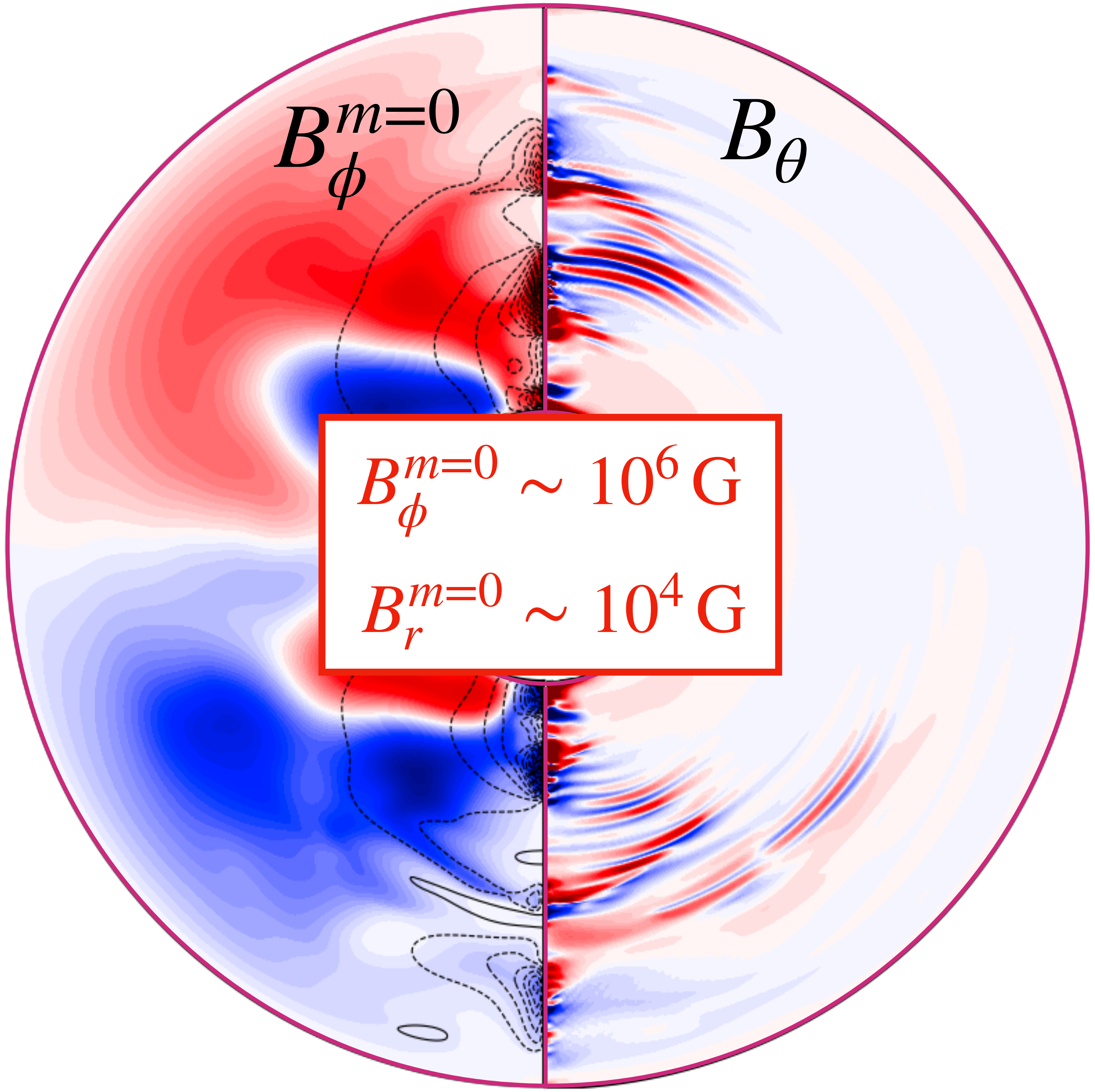
First anelastic simulations

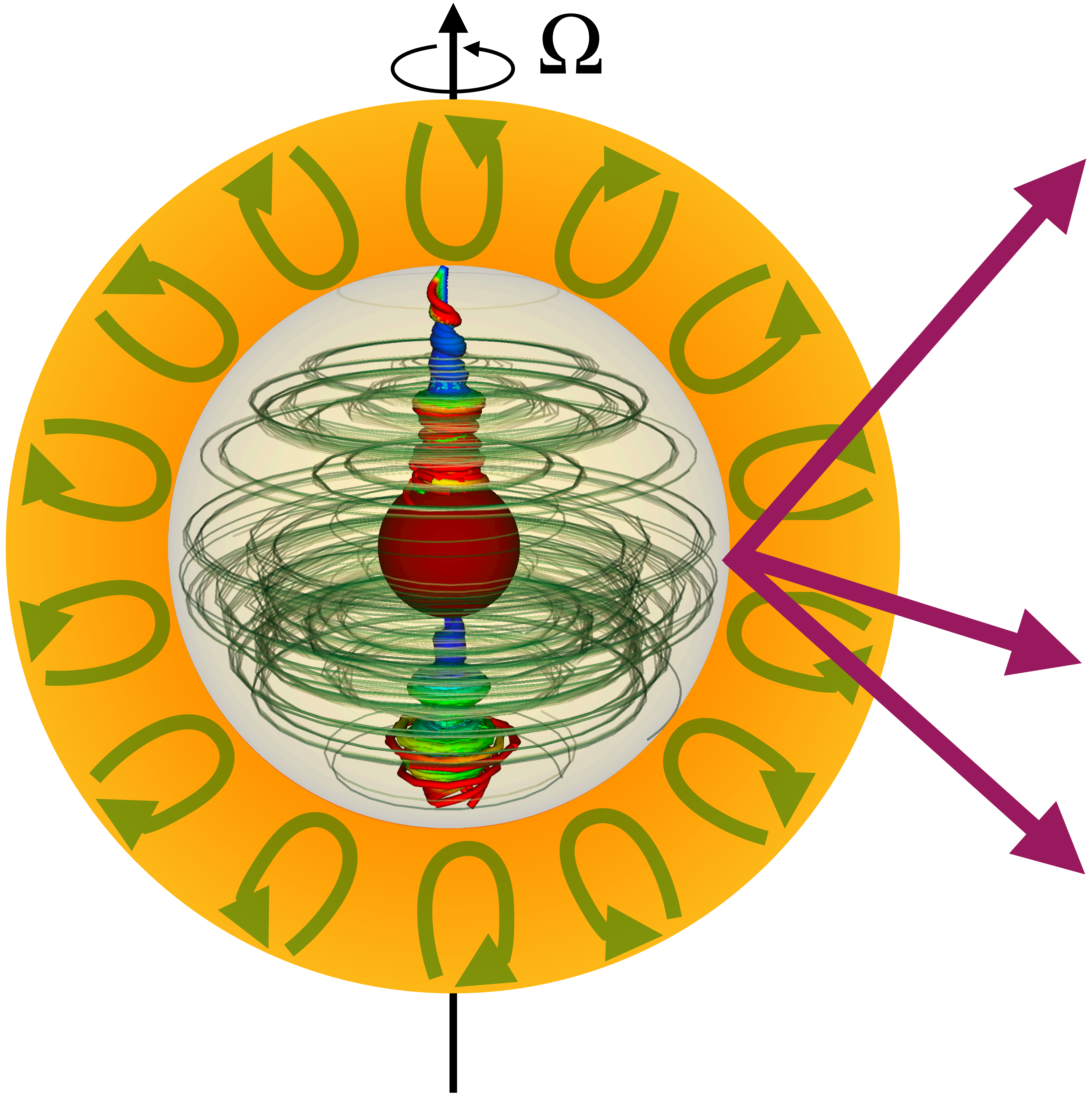
Anelastic approximation:
- **Realistic** density profile
- Flow only produce **perturbations of a reference state**

- Realistic structures
- Impact of a density gradient

- Numerically costly
- Reference state is stationary







New information thanks to 3D models:

- Tayler instability-driven dynamo active at **large** N/Ω
  $\Longrightarrow$ Critical $Rm_{\mathrm{turb}} \sim 10$
- **Toroidal** component ($\ell = 2, m = 0$) dominates
- Radial field near the **polar axis** ($\ell = 2, m = 0$)

Perspectives

Physics:

- Anelastic simulations with realistic backgrounds
- Characterise the nonlinear saturation
- Study the subcritical transition between both branches

Astrophysics:

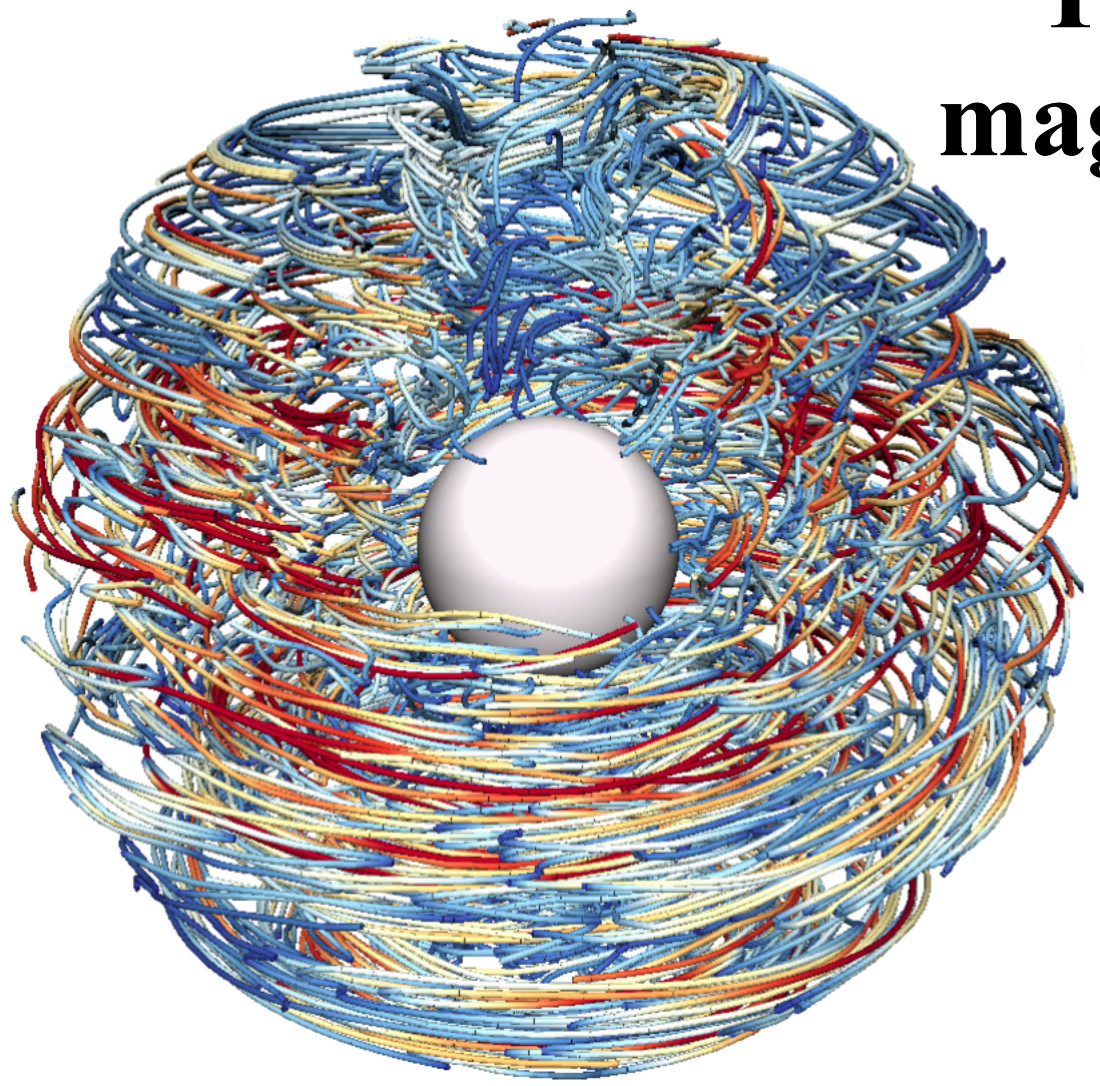
- **Signature in the astero. signal**
- **Prescriptions for 1D stellar evolution models**

Additional material

Origin of the magnetic field

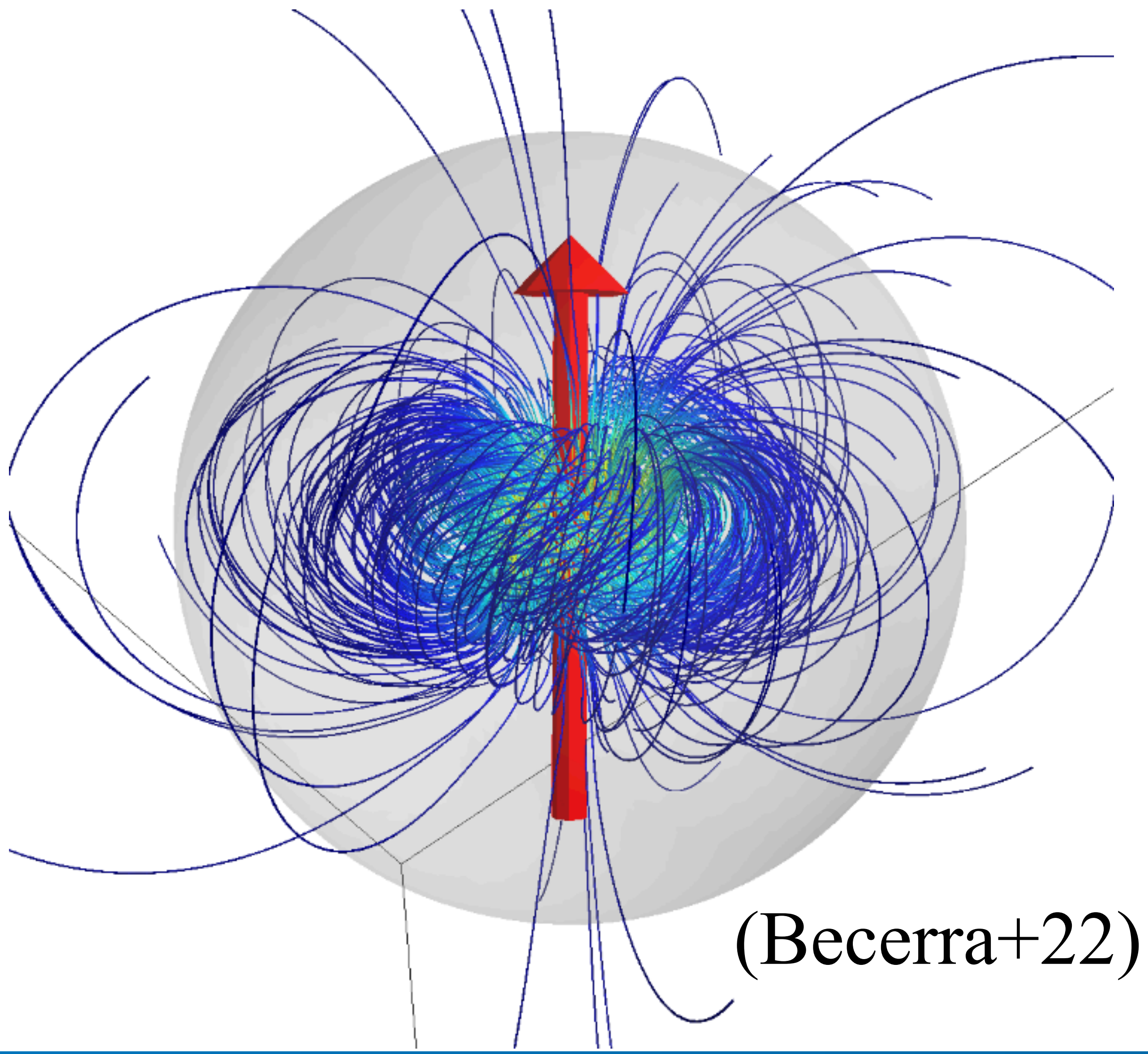
Fossil field origin

Turbulent magnetic field



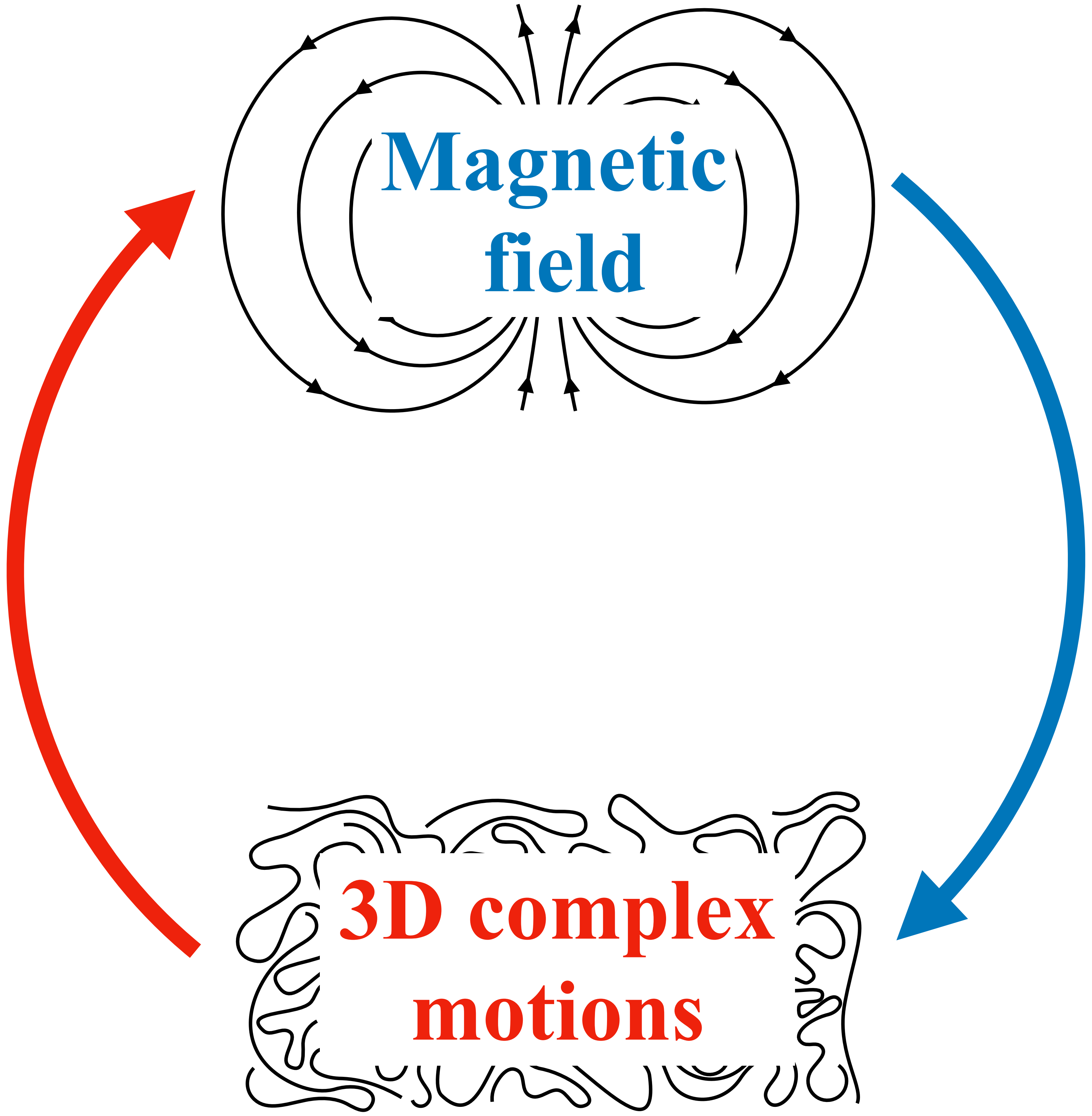
(Reboul-Salze+21)

Stable configuration



(Becerra+22)

Dynamo action

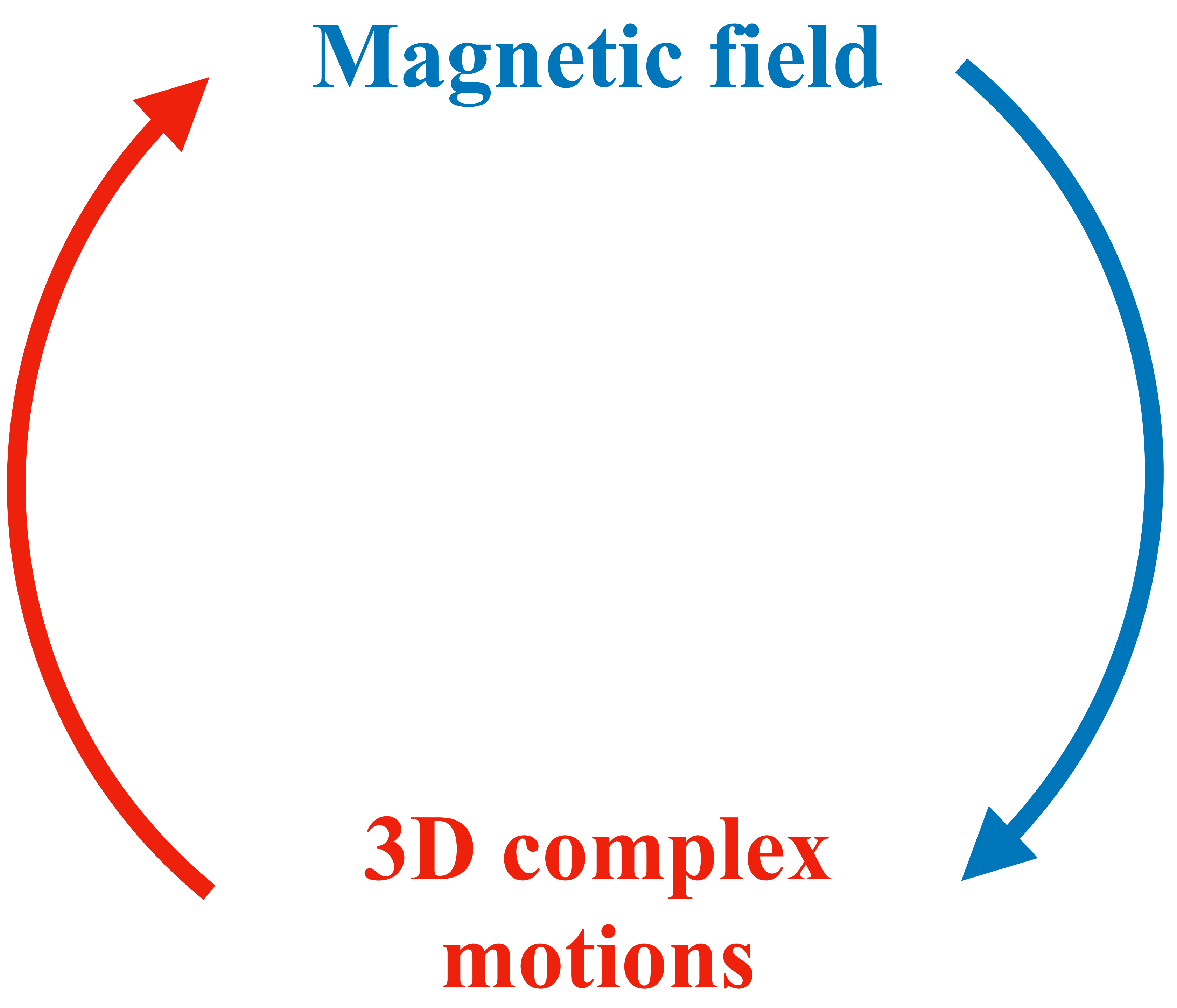


Diversity of motion sources

=

Different dynamo mechanisms

Astrophysical dynamos

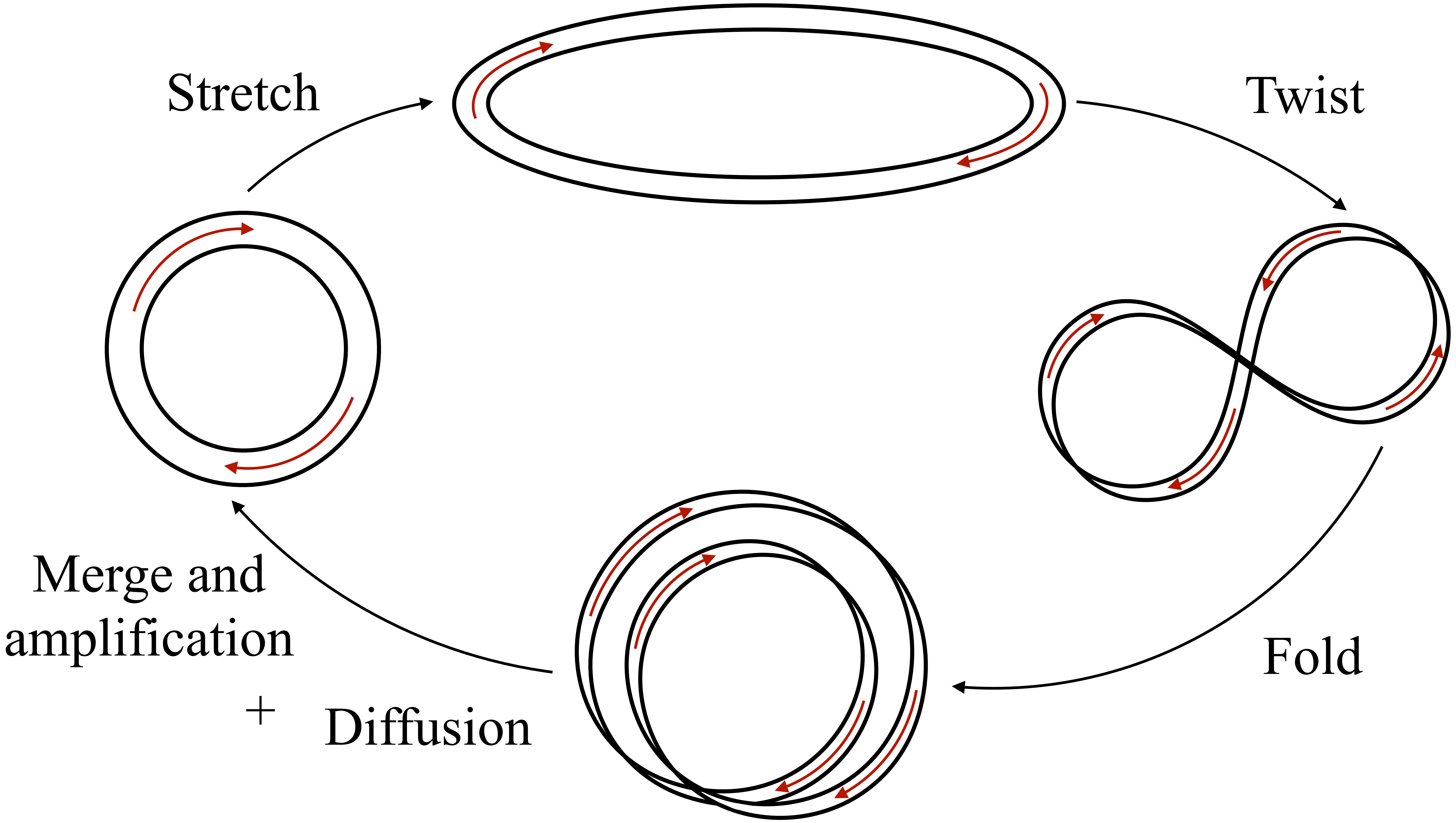


Diversity of motion sources

=

Different dynamo mechanisms

Vainshtein & Zel'dovich fast dynamo process



Astrophysical dynamos

Magnetic field

« Self-exciting fluid dynamos » by K. Moffatt & E. Dormy

Reviews by F. Rincon 2019 and S. Tobias 2021

3D complex motions

Diversity of motion sources

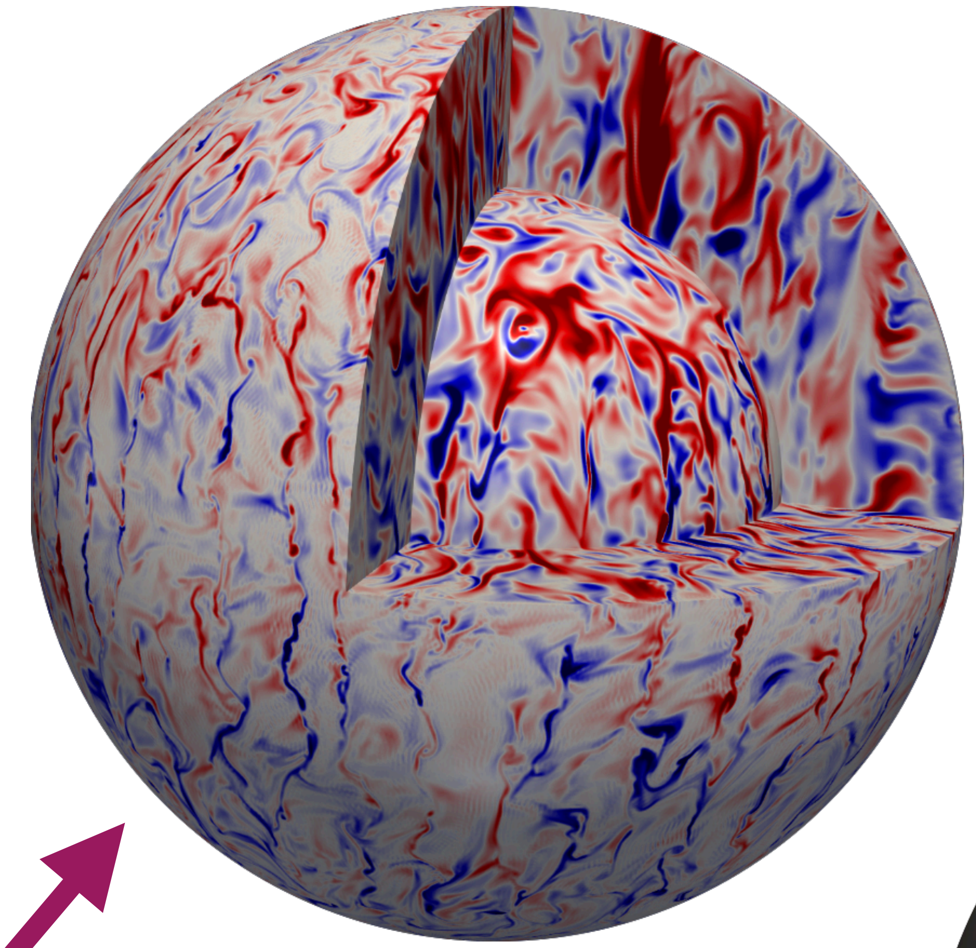
=

Different dynamo mechanisms

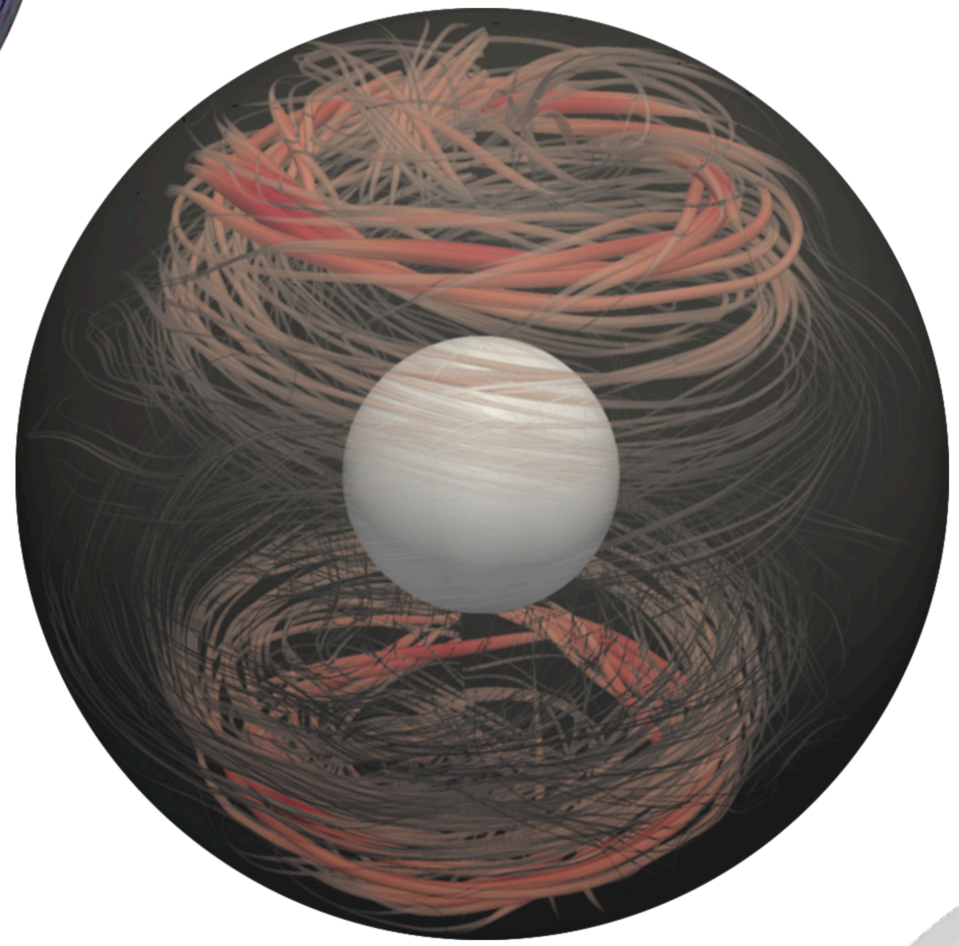
Convection

MRI

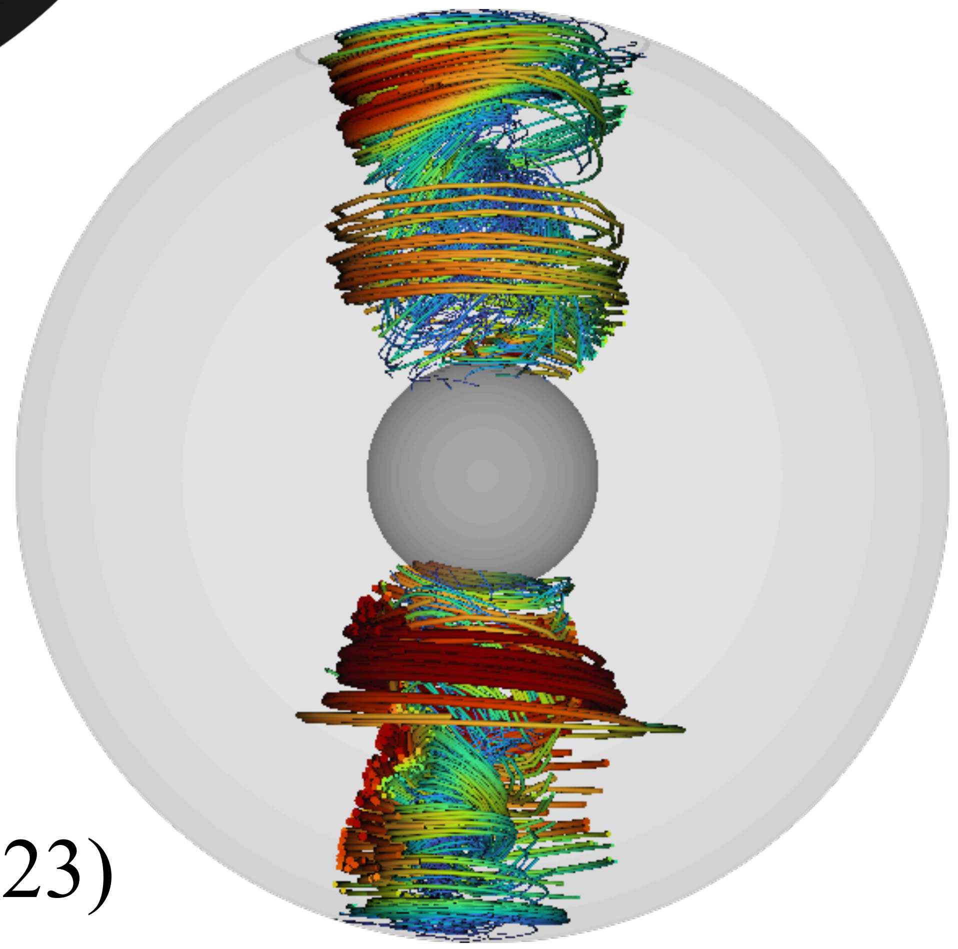
Tayler instability



(Zaire+22)



(Jouve+15)



(Barrère+23)

Main approximations of the modeling

Filter out acoustic waves

Boussinesq approximation:
- **Uniform** density profile
- Density perturbations **ONLY** in buoyancy term

Anelastic approximation:
- **Realistic** density profile
- Flow only produce **perturbations of a reference state**

Dimensionless parameters and stratification

Ekman number $E \equiv \dfrac{\nu}{d^2\Omega_o}$

Rayleigh number $Ra \equiv \dfrac{N^2 d^4}{\nu\kappa}$

Thermal Prandtl number $Pr \equiv \dfrac{\nu}{\kappa}$

Magnetic Prandtl number $Pm \equiv \dfrac{\nu}{\eta}$

Same diffusivities for S and Y_e

Proto-neutron stars

$T \longrightarrow \Theta$

$$N^2 \equiv -\frac{g}{\rho}\left(\frac{\partial\rho}{\partial S}\bigg|_{P,Y_e}\frac{dS}{dr} + \frac{\partial\rho}{\partial Y_e}\bigg|_{P,S}\frac{dY_e}{dr}\right)$$

Radiative zones

$$N^2 \equiv -\frac{g}{\rho}\left(\frac{\partial\rho}{\partial S}\bigg|_{P,\mu}\frac{dS}{dr} + \cancel{\frac{\partial\rho}{\partial \mu}\bigg|_{P,S}\frac{d\mu}{dr}}\right)$$

Forcing of differential rotation

Forcing on the boundaries
- Impose fixed Ω_i and Ω_o
- Spherical-Couette configuration
- Instabilities due to the set-up

Volumetric forcing
- $\mathbf{v} = \mathbf{u} + v_\mathrm{f}\mathbf{e}_\phi$ with $v_\mathrm{f} = r\sin\theta\Omega_\mathrm{f}$ stationary field
- **Body force**: $\mathbf{f} = -u_\phi^{m=0}/t_\mathrm{f}$
- **Forcing timescale**: $t_\mathrm{f} \sim 10 \times t_\Omega$

Volumetric forcing in MagIC

Separation of variables: $\vec{V} = \vec{U} + \vec{V_f}$

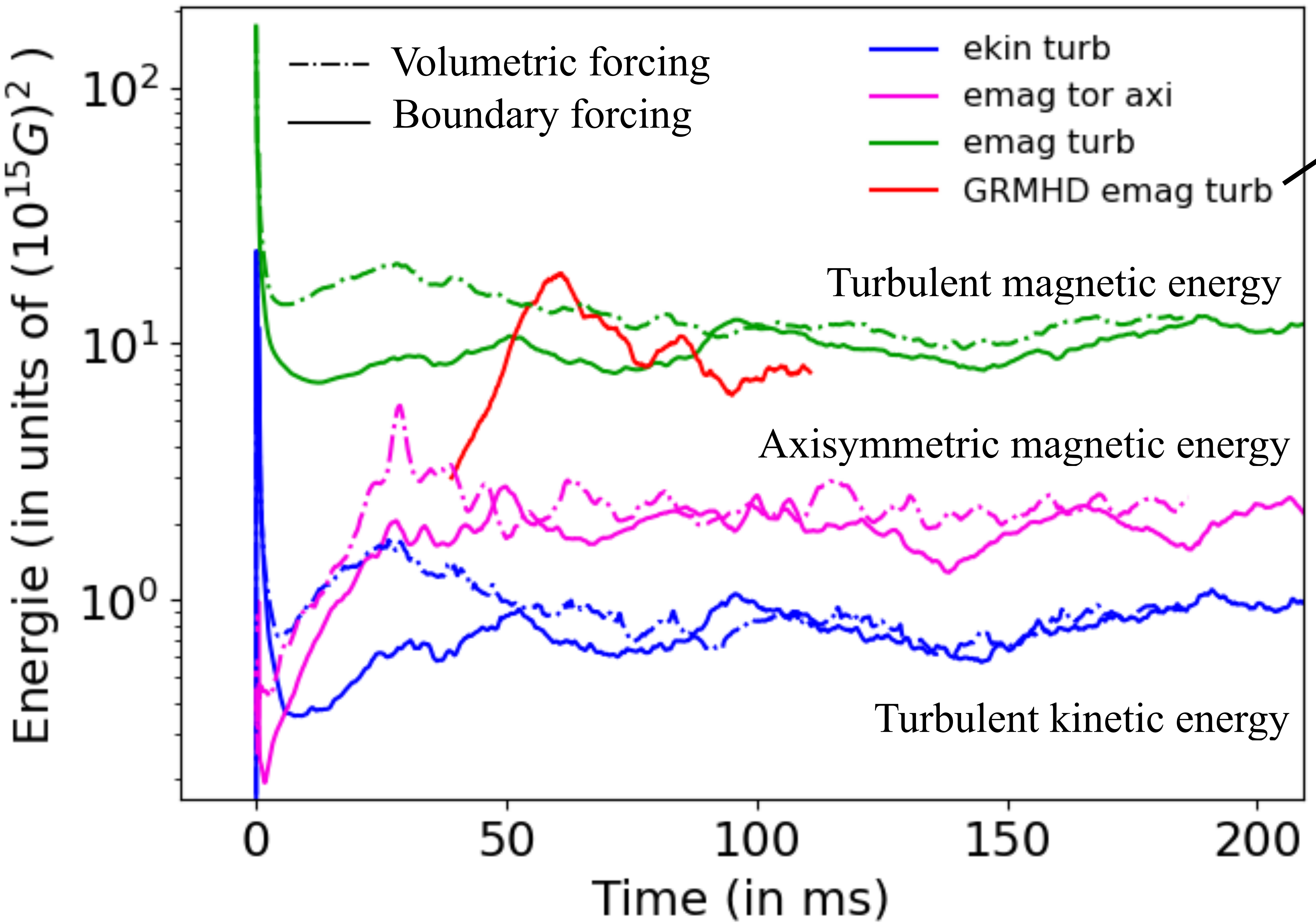
$$
\begin{cases}
\vec{\nabla} \cdot \left(\bar{\rho}\vec{U}\right) = 0 \quad \text{et} \quad \vec{\nabla} \cdot \vec{B} = 0, \\[2ex]
\dfrac{\partial U}{\partial t} + \left(\left(\vec{U} + \vec{V_f}\right) \cdot \vec{\nabla}\right)\left(\vec{U} + \vec{V_f}\right) + \overset{\textbf{Coriolis}}{2\Omega_0 \vec{e}_z \times \left(\vec{U} + \vec{V_f}\right)} = \overset{\textbf{Pressure}}{-\vec{\nabla}\left(\dfrac{\Pi'}{\bar{\rho}}\right)} + \overset{\textbf{Buoyancy}}{\dfrac{g_0 r_0^2}{C_p r^2} S' \vec{e}_r} \\
\qquad \overset{\textbf{“Dissipation” Forcing}}{-\tau U_{\phi,m=0}} + \overset{\textbf{Lorentz}}{\dfrac{1}{\mu_0 \bar{\rho}}\left[\left(\vec{\nabla} \times \vec{B}\right) \times \vec{B}\right]} + \overset{\textbf{Viscosity}}{\dfrac{\nu}{\bar{\rho}} \vec{\nabla} \cdot \vec{\vec{\sigma}}}, \\[2ex]
\dfrac{\partial \vec{B}}{\partial t} = \overset{\textbf{Induction}}{\vec{\nabla} \times \left[\left(\vec{U} + \vec{V_f}\right) \times \vec{B}\right]} + \overset{\textbf{Resistivity}}{\eta \vec{\nabla}^2 \vec{B}}, \\[2ex]
\bar{\rho}\bar{T}\left(\dfrac{\partial S'}{\partial t} + \overset{\textbf{Advection}}{\left(\left(\vec{U} + \vec{V_f}\right) \cdot \vec{\nabla}\right) S'} + \left(U_r + V_f\right)\dfrac{\mathrm{d}\bar{S}}{\mathrm{d}r}\right) = \overset{\textbf{Heat flux}}{\kappa \vec{\nabla} \cdot \left(\bar{\rho}\bar{T}\vec{\nabla} S'\right)} + \overset{\textbf{Heating}}{\nu Q_\nu + \dfrac{\eta}{\mu_0^2}\left(\vec{\nabla} \times \vec{B}\right)^2}
\end{cases}
$$

Implemented in MagIC by A. Reboul-Salze

→ More efficient forcing than just forcing at the outer boundaries

In the following $\tau^{-1} \sim 10\, P_{\overline{\Omega}}$

Boundary vs. volumetric forcing: case for MRI



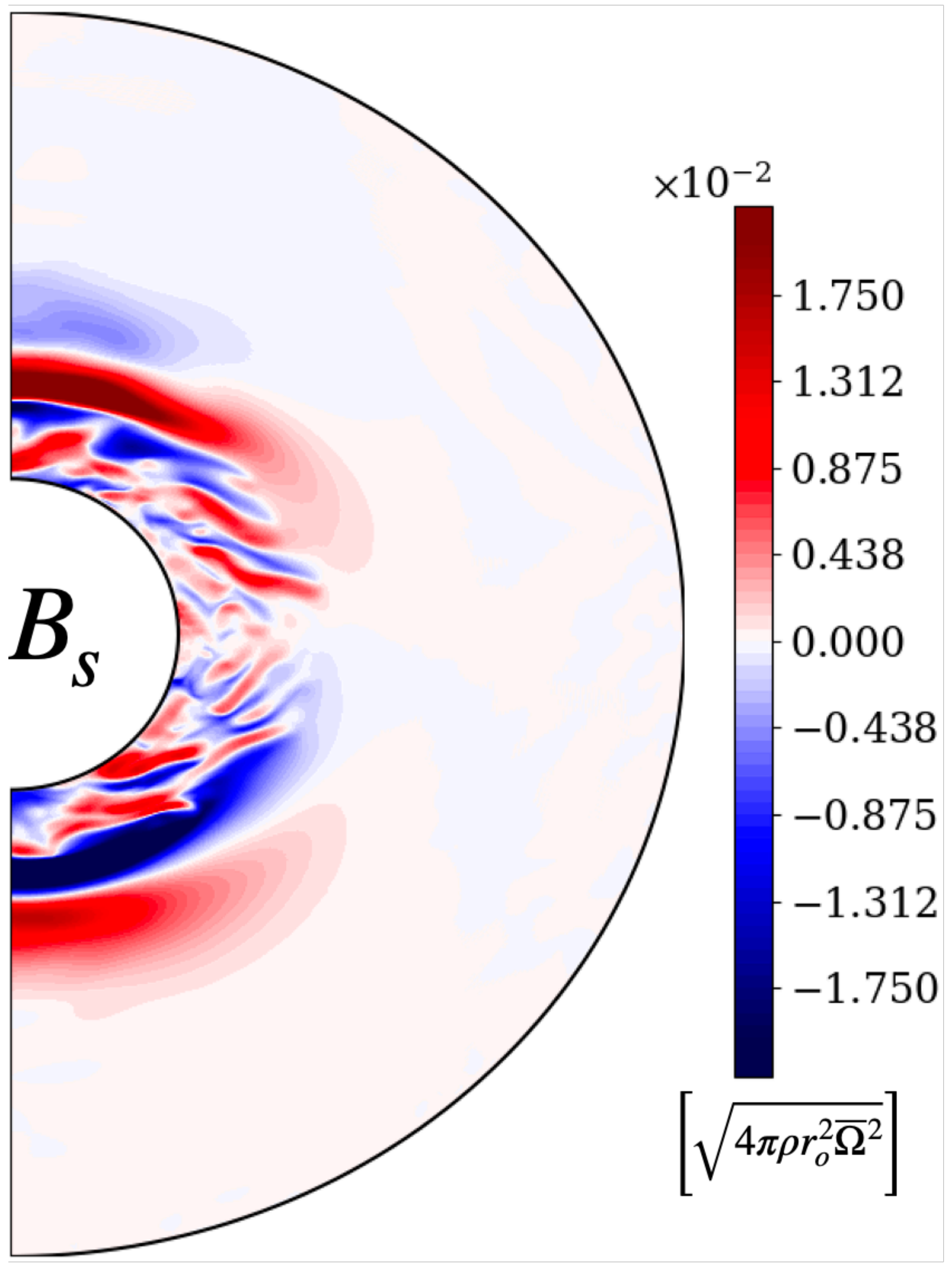
GRMHD energy with the inertial magnetic field

B ~ 3×10^{15}

B ~ 1.5×10^{15} G

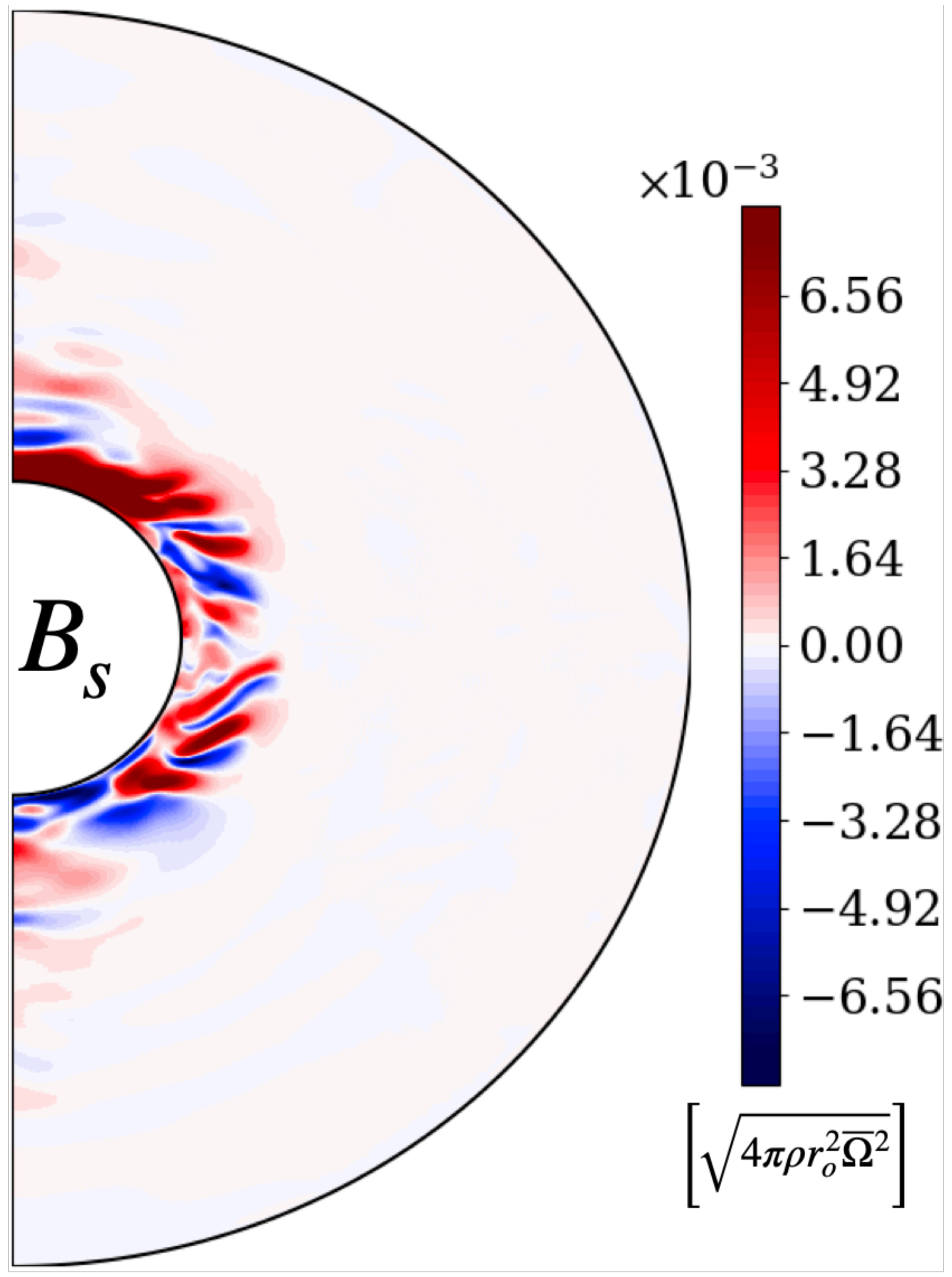
$$\boxed{\tau^{-1} \sim 10\, P_{\overline{\Omega}}}$$

Two different dynamos



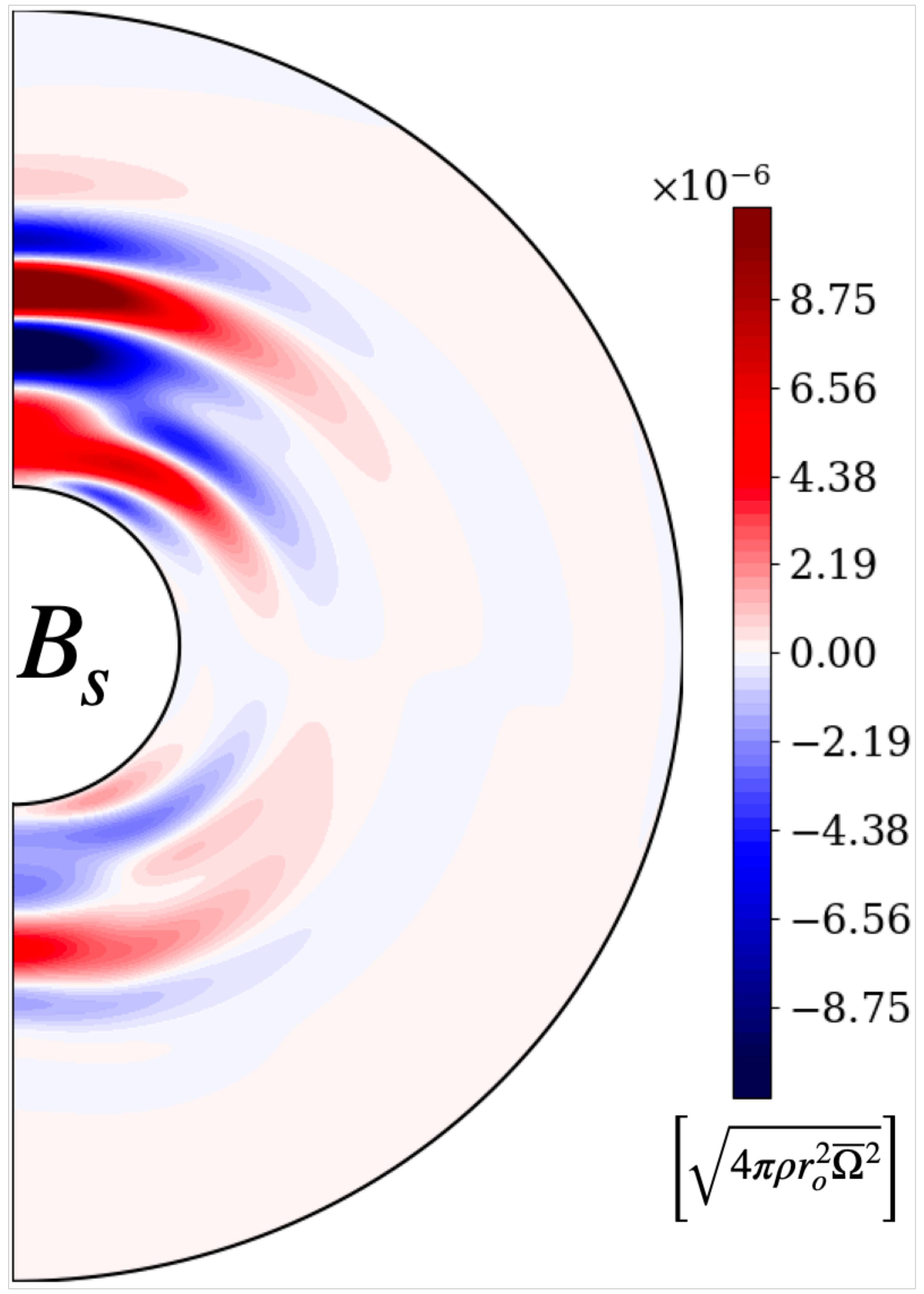
$q = -1$

$B_\phi^{m=0} \propto 1/r$



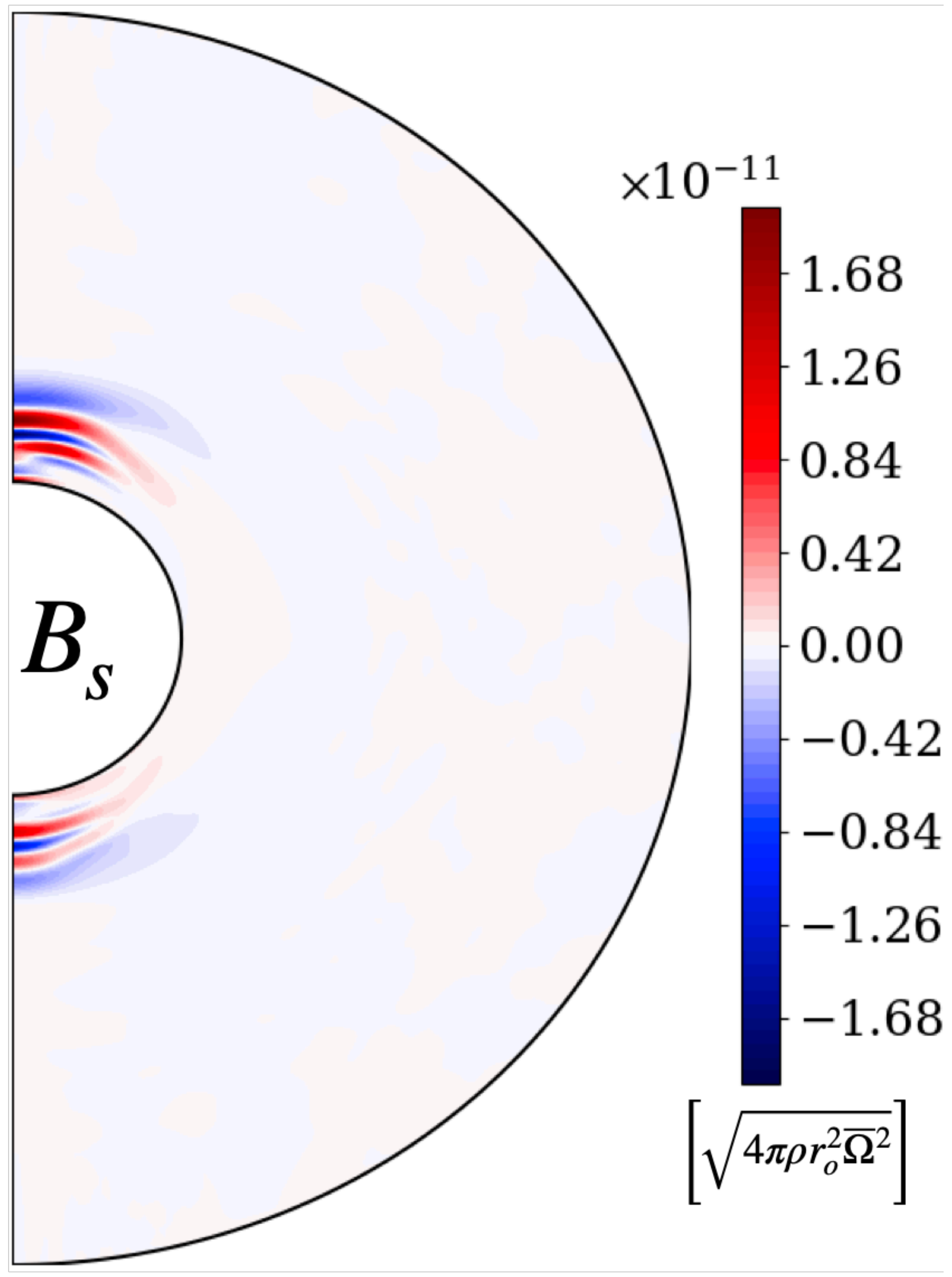
$q = -1$

$B_\phi^{m=0} \propto 1/r^2$



$q = 0$

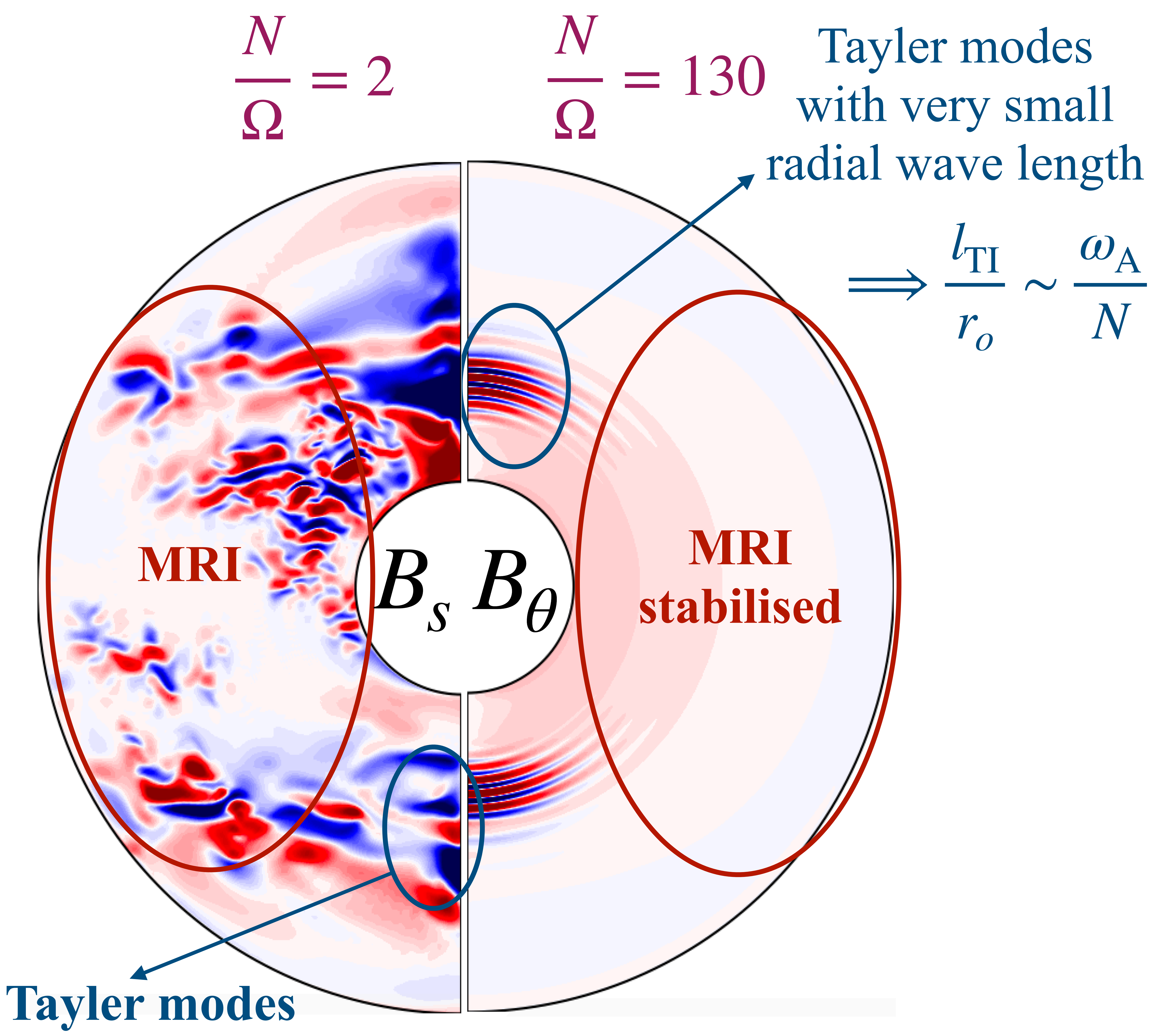
$B_\phi^{m=0} \propto 1/r$

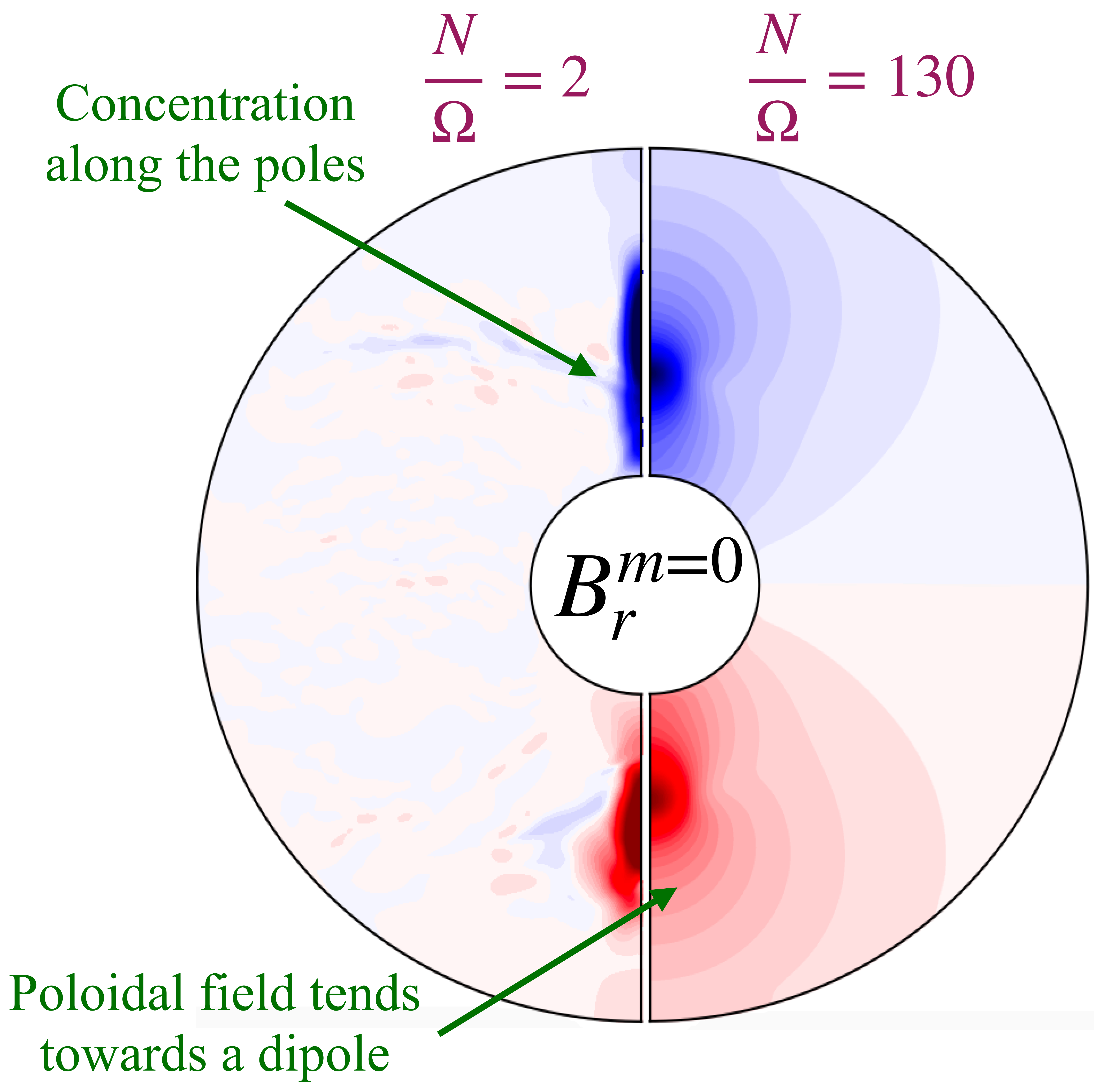


$q = 0$

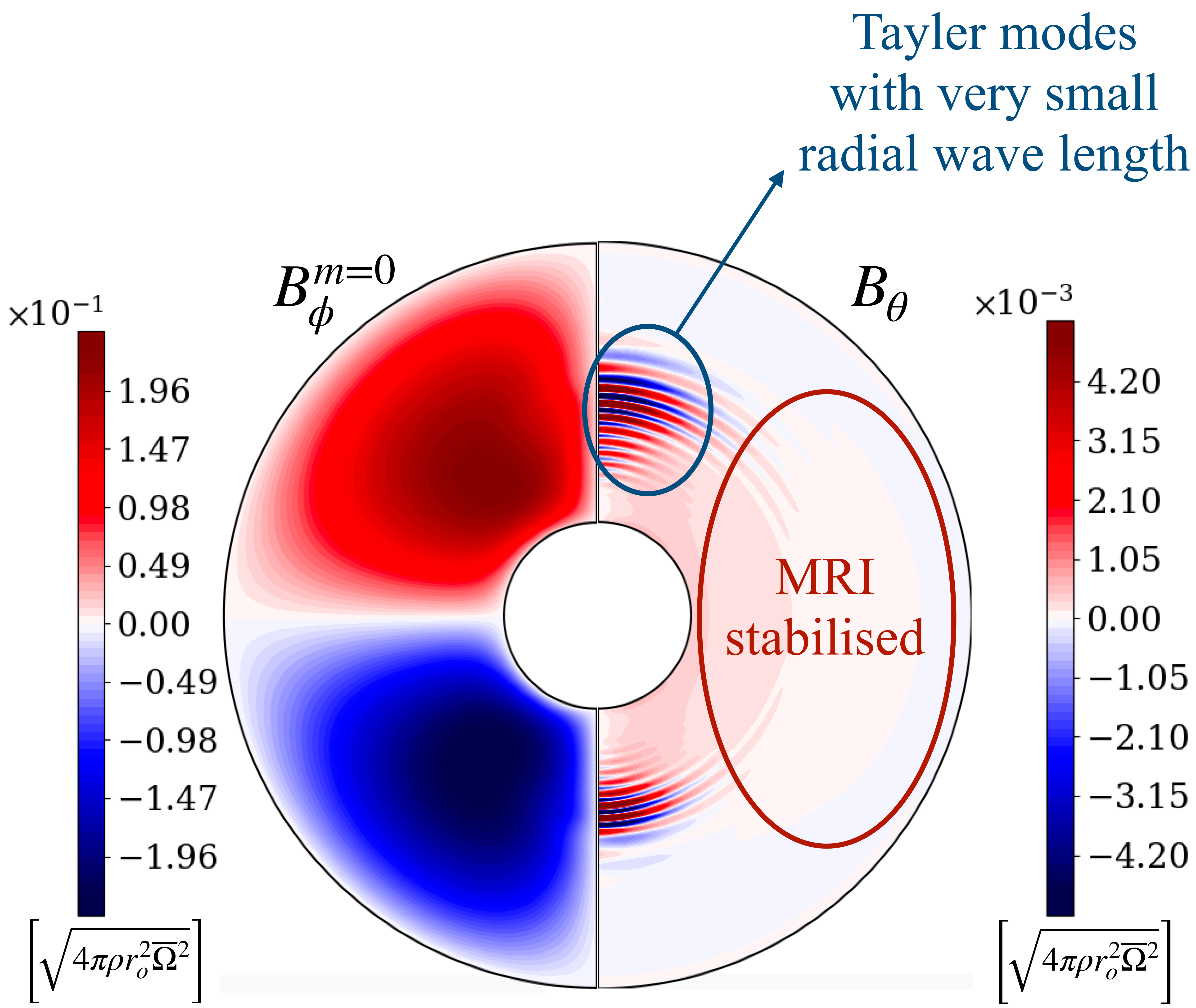
$B_\phi^{m=0} \propto 1/r^2$

Dipolar branch in a large range of stratifications



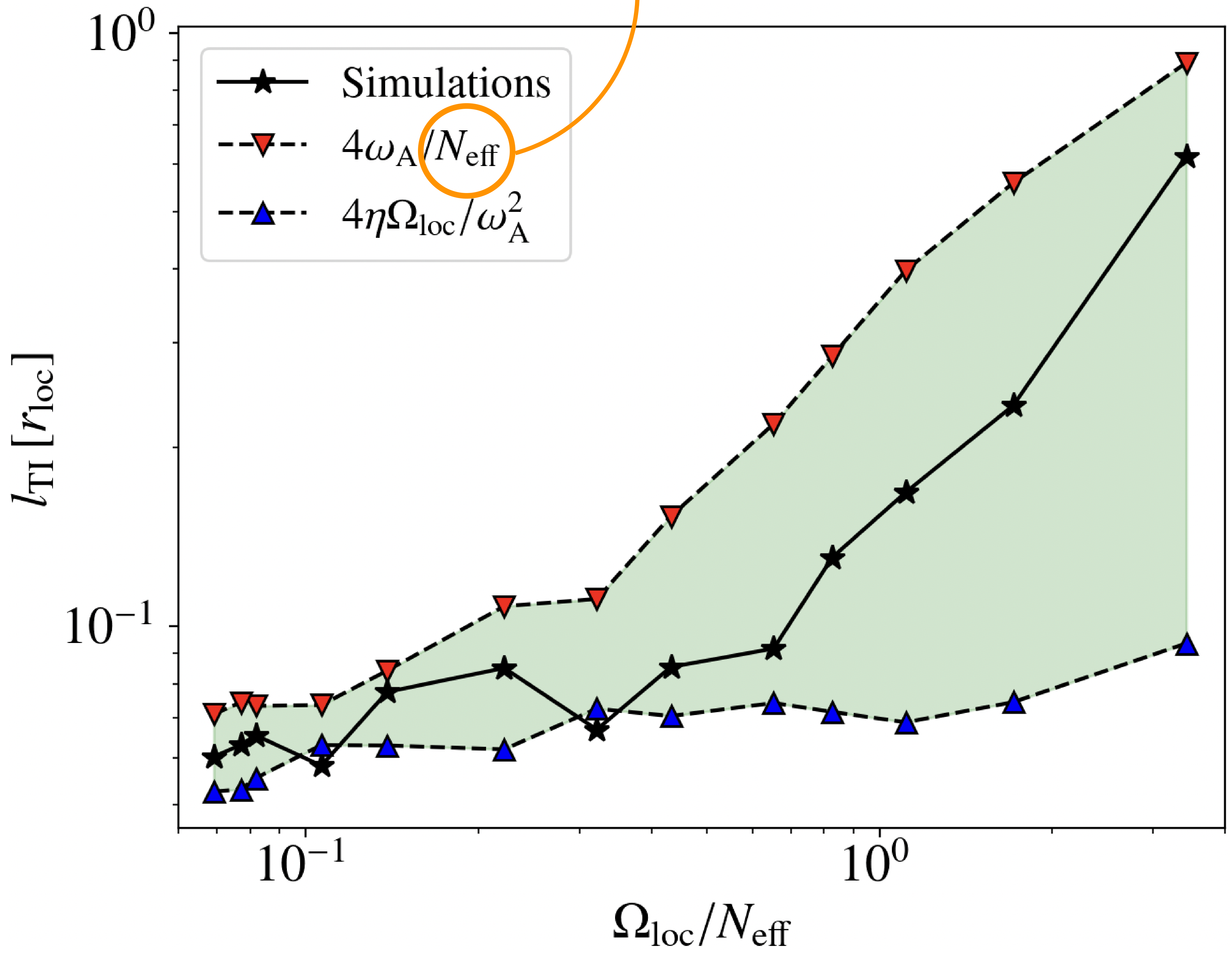


Impact on the Tayler mode radial length scale

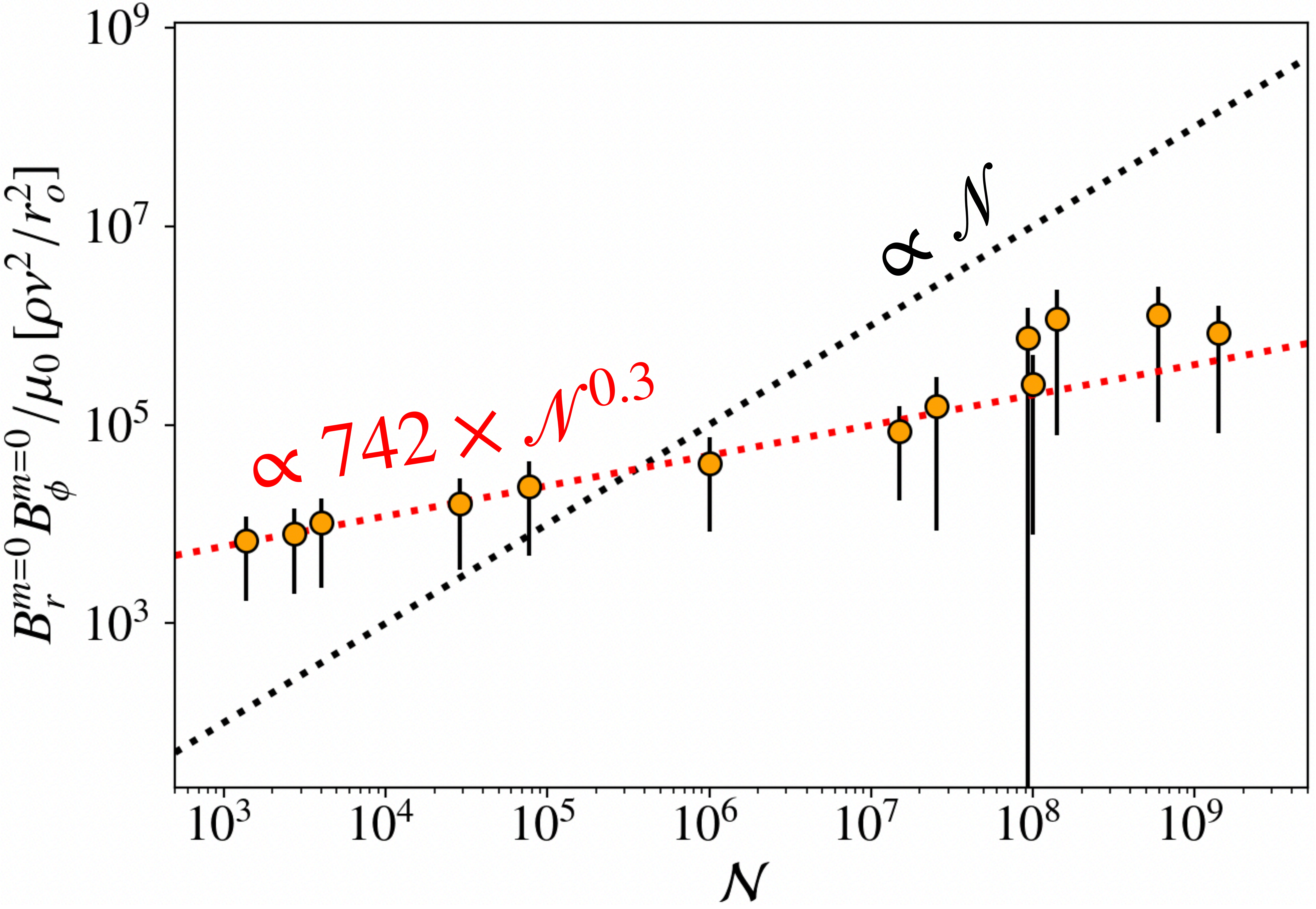


Effective BV frequency

$$N_{\text{eff}}^2 = \frac{\eta}{\kappa} N^2$$



Scaling Maxwell torque as in Petitdemange+23



$$\mathcal{N} = \beta q^3 \frac{r_o^4 \overline{\Omega}^2}{\nu^2} \left(\frac{\overline{\Omega}}{N} \right)^4$$

with $q = \dfrac{u_\phi^{\mathrm{loc}}}{l_{\mathrm{TI}} \overline{\Omega}}$ and $\beta = 0.1$

Comparison with Petitdemange et al. 2023, 2024

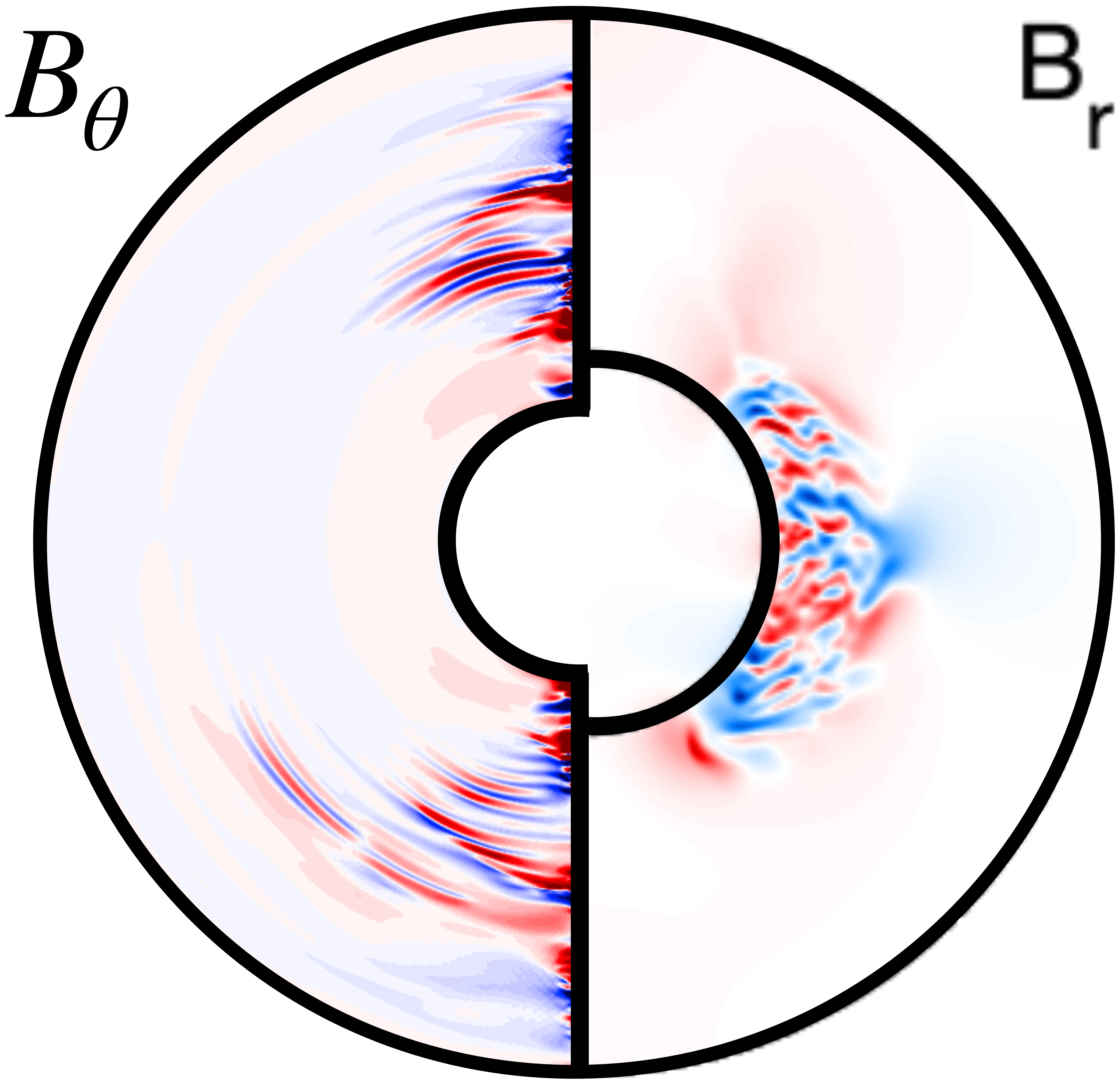
This study	Petitdemange et al. 2023, 2024:

Setup differences:

▸ **Volum. forcing**	▸ **Boundary forcing**

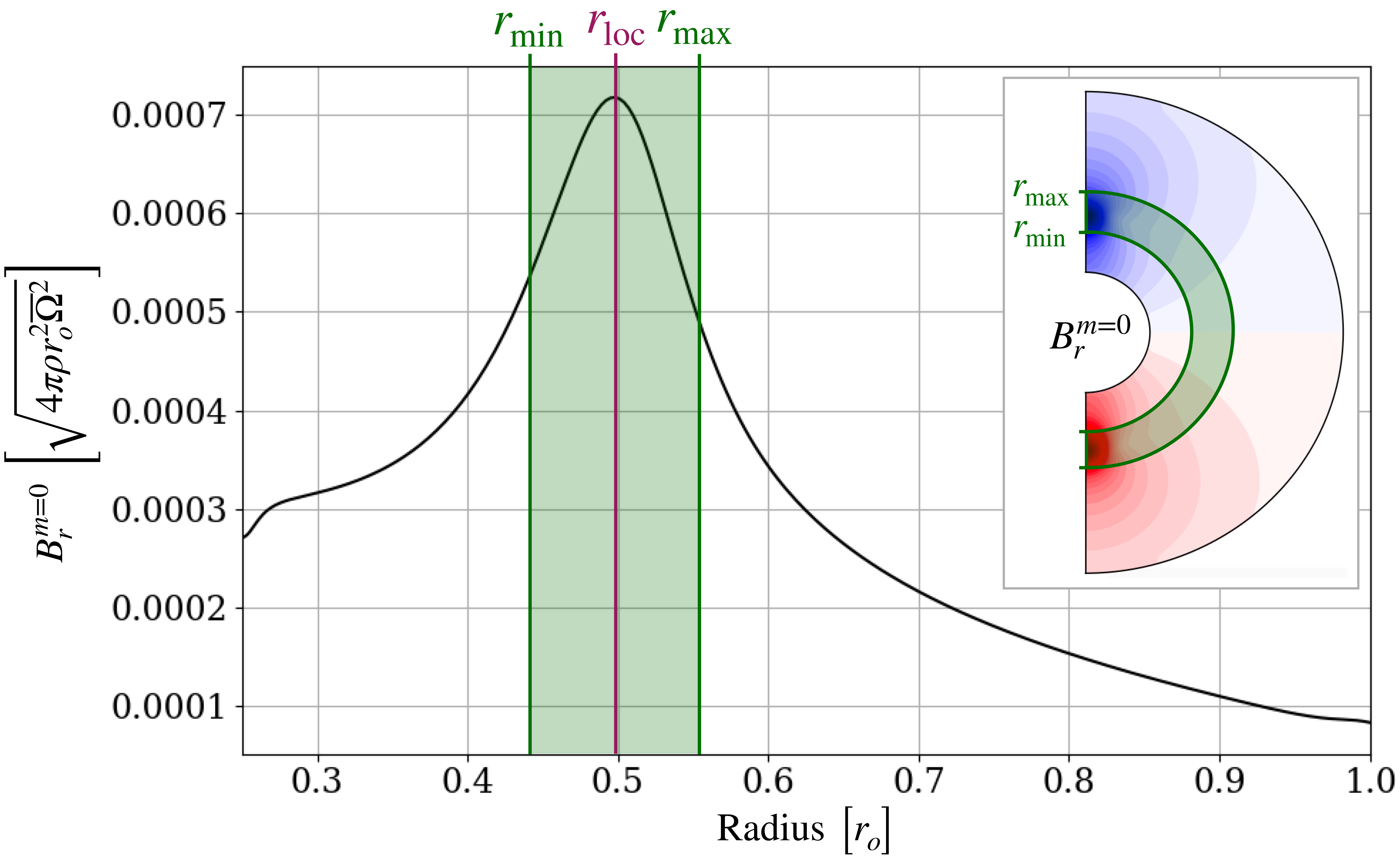
Mag. Field differences:

▸ Near the **poles**	▸ Near the **equator**
▸ **More efficient tranport**	▸ **Spruit 2002**’s scaling law



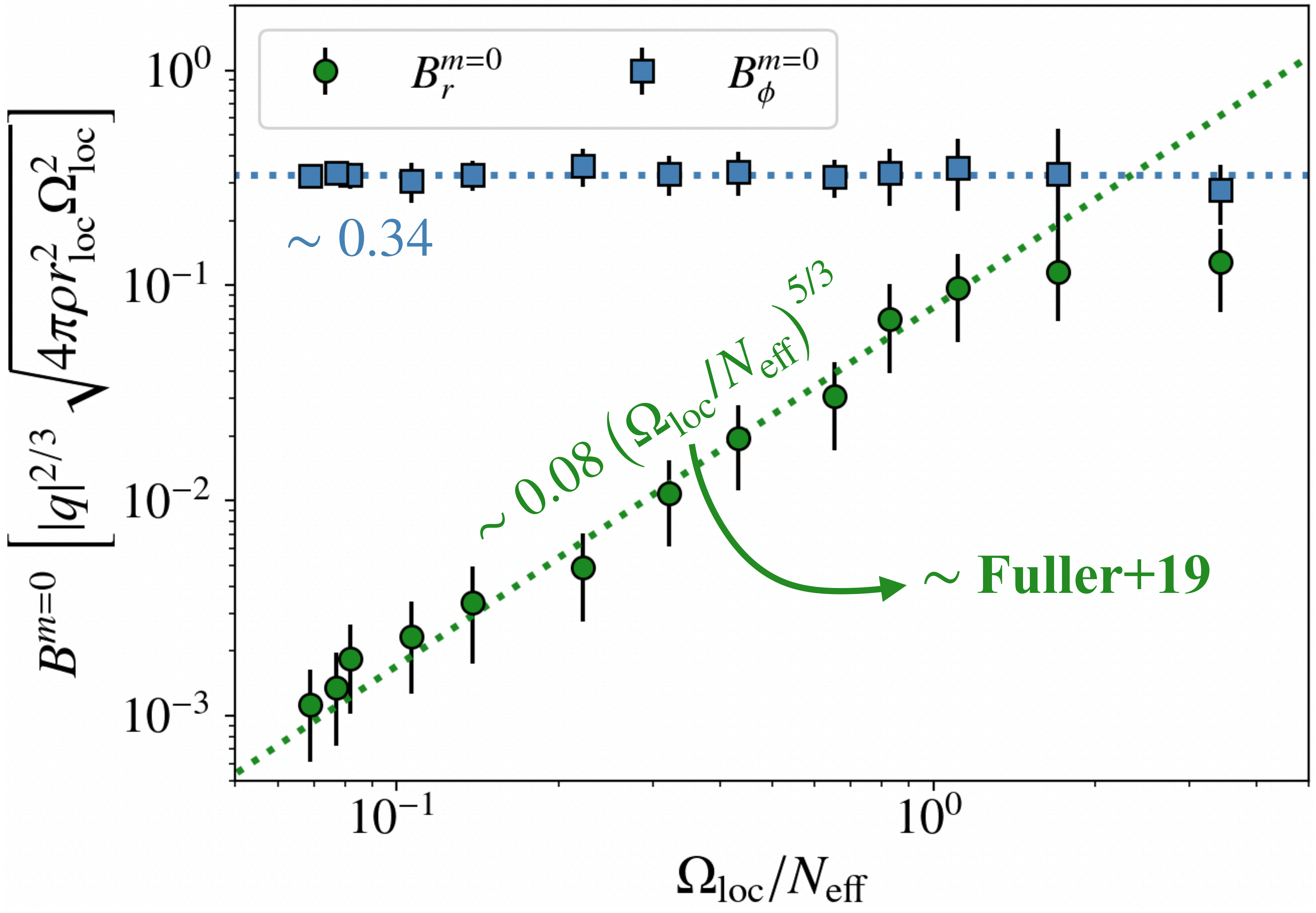
This study Petitdemange et al. 2024

Radial profile after latitudinal and time average

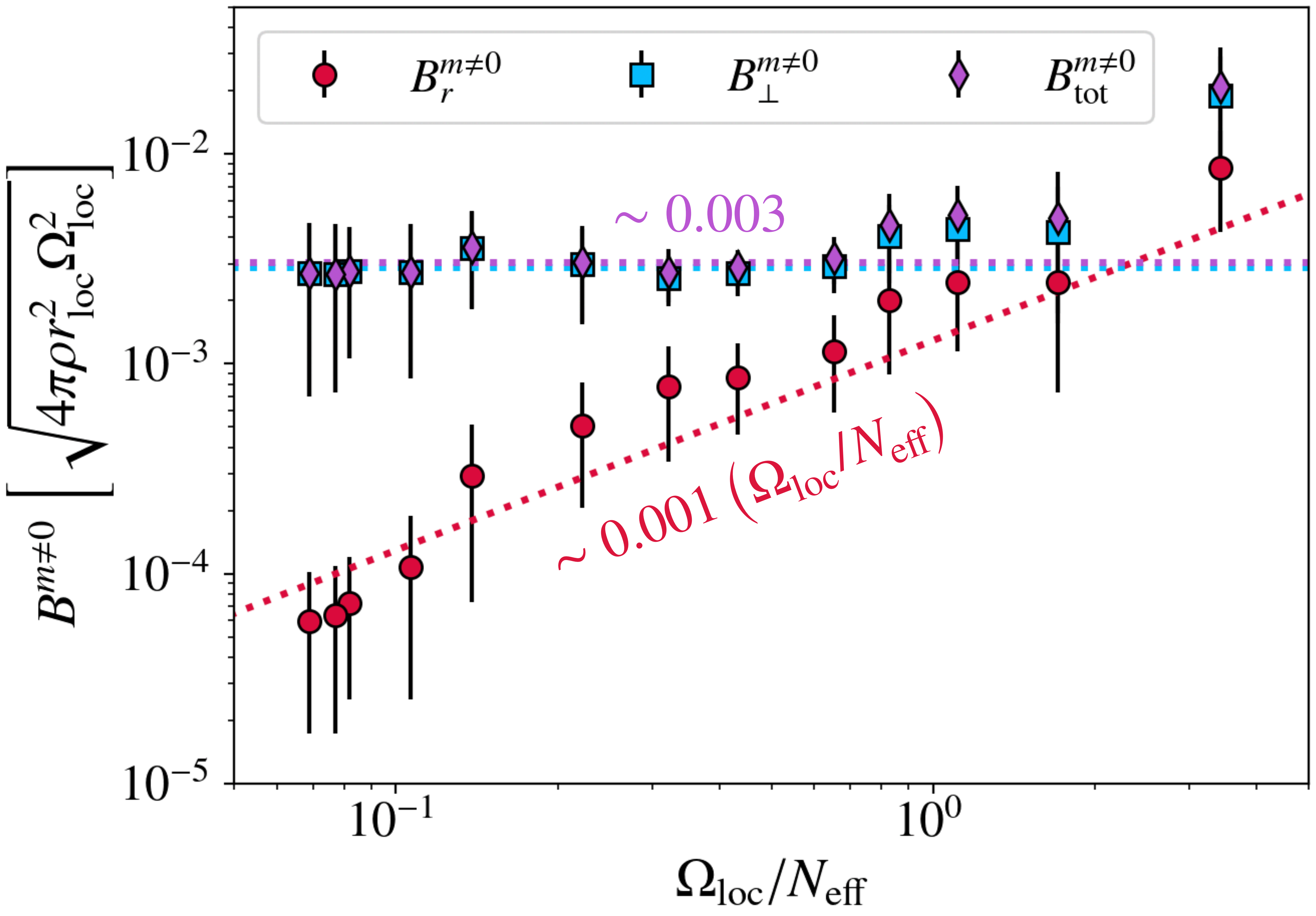


Magnetic field scaling laws

Large-scale magnetic fields



Turbulent magnetic fields

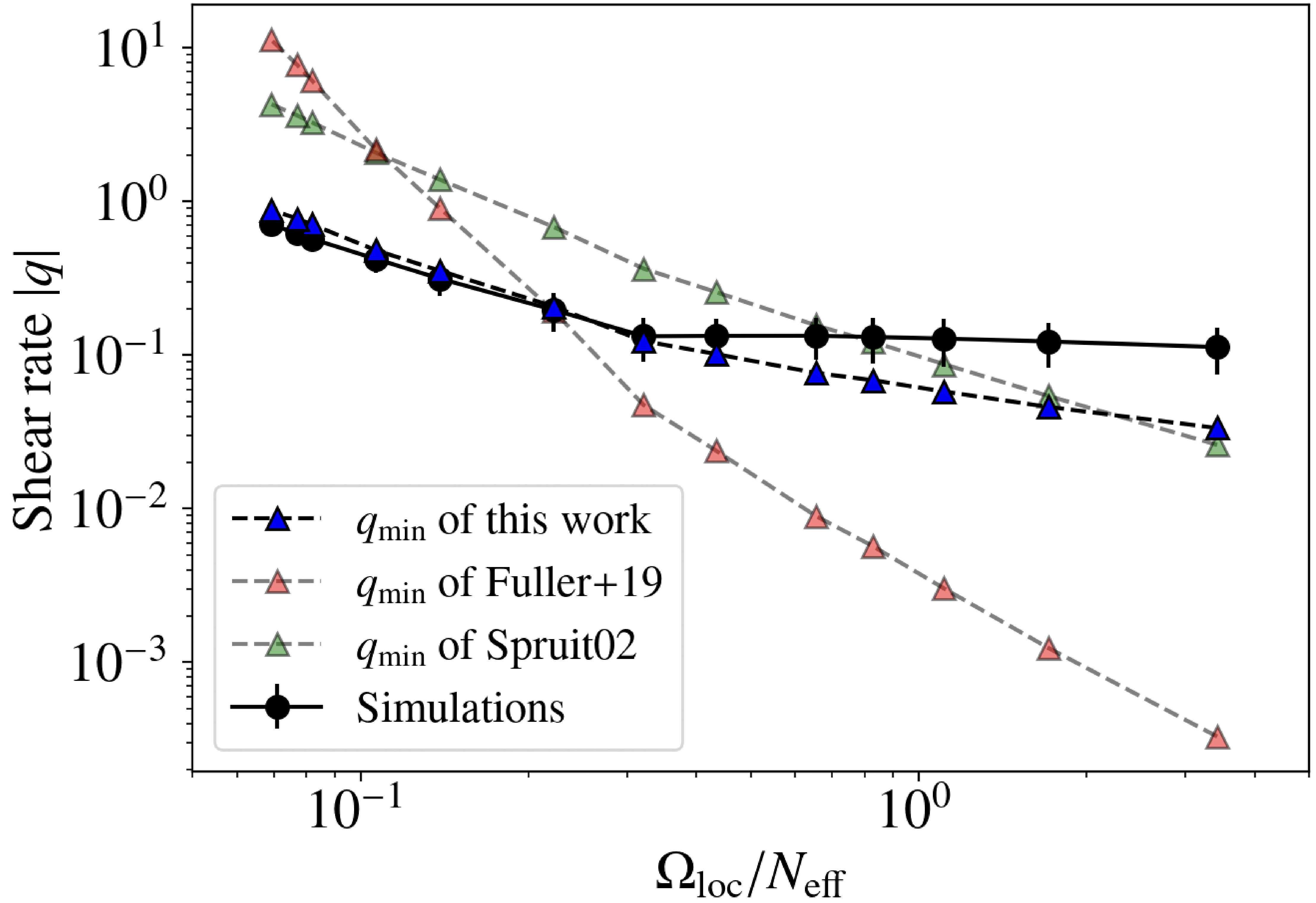


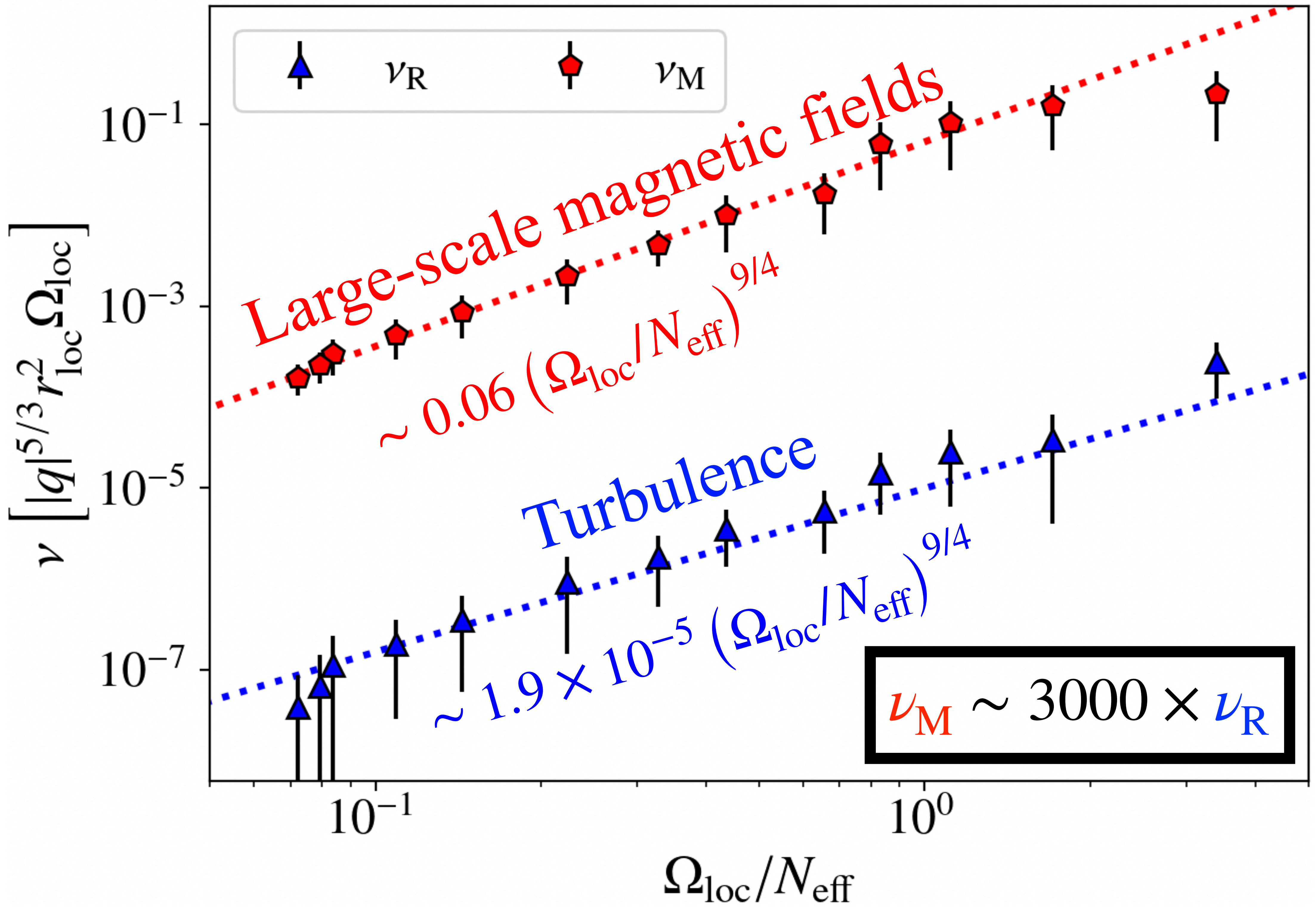
New expression of $q_{\min}$:

$$q_{\min} \sim 5.2 \left(\frac{N_{\mathrm{eff}}}{\Omega_{\mathrm{loc}}}\right)^{3/4} \left(\frac{\eta}{r_{\mathrm{loc}}^2 \Omega_{\mathrm{loc}}}\right)^{3/8}$$

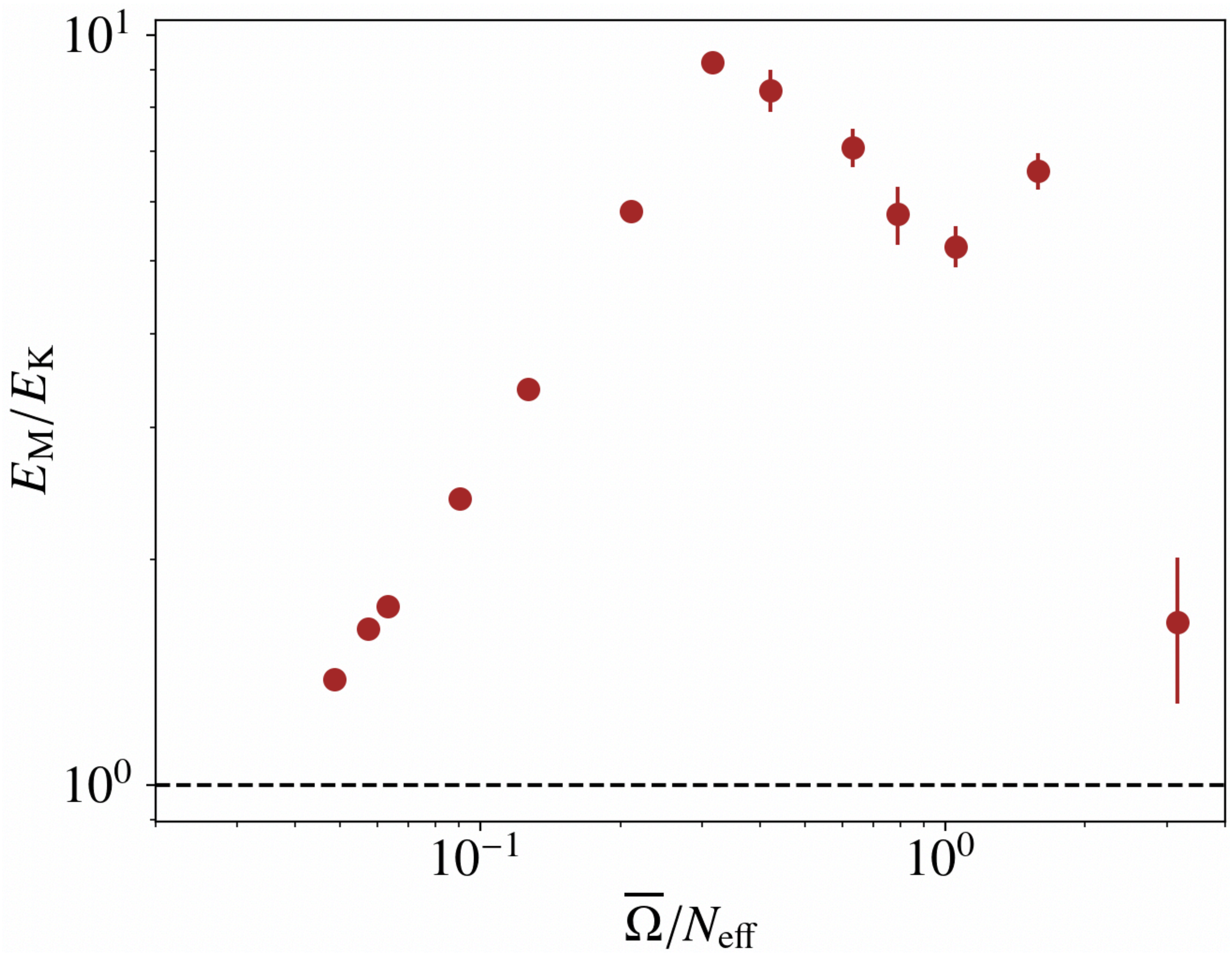
Fuller+19:

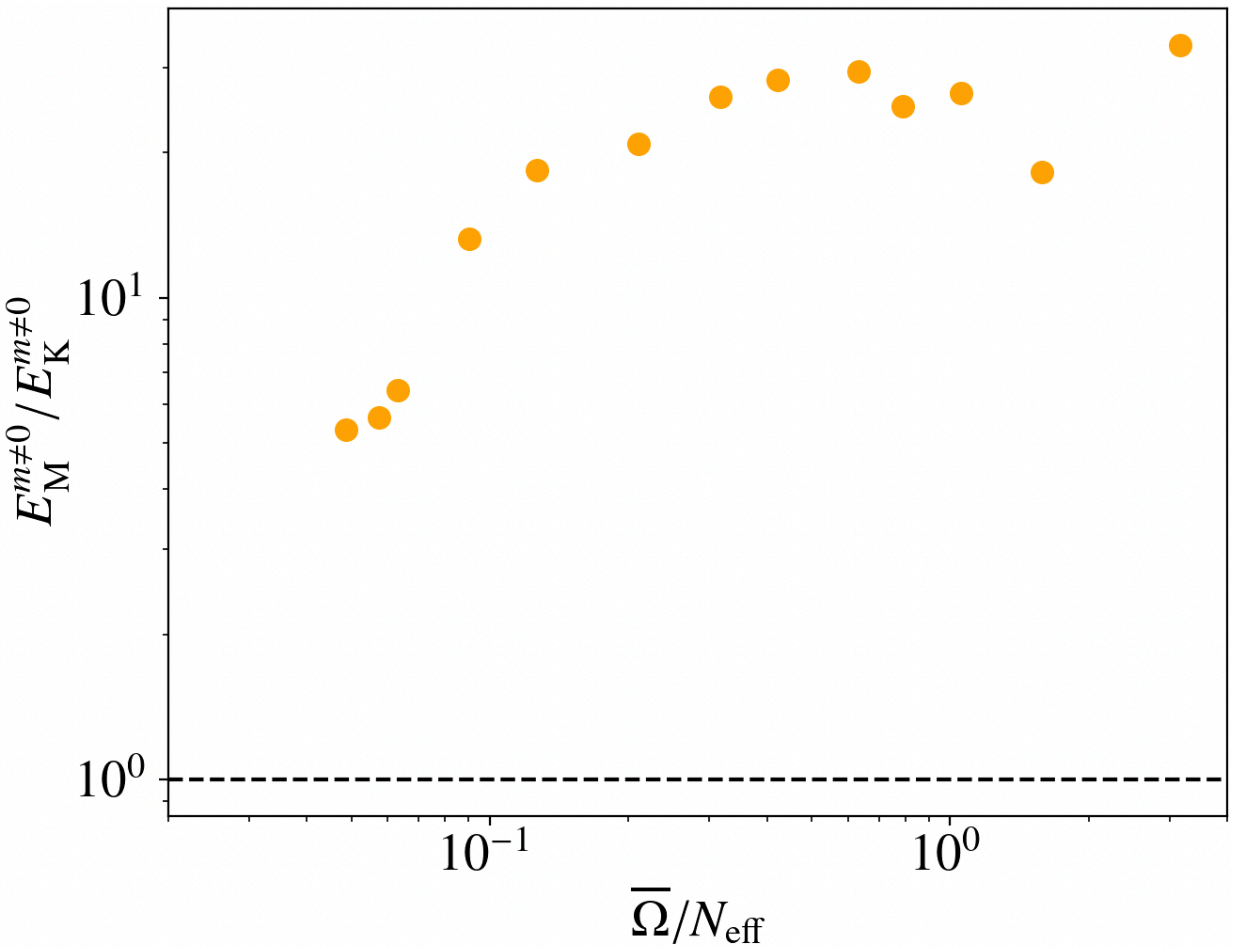
$$\nu_{\mathrm{M}} \sim 0.06\, q r_{\mathrm{loc}}^2 \Omega_{\mathrm{loc}} \left(\Omega_{\mathrm{loc}}/N_{\mathrm{eff}}\right)^2$$

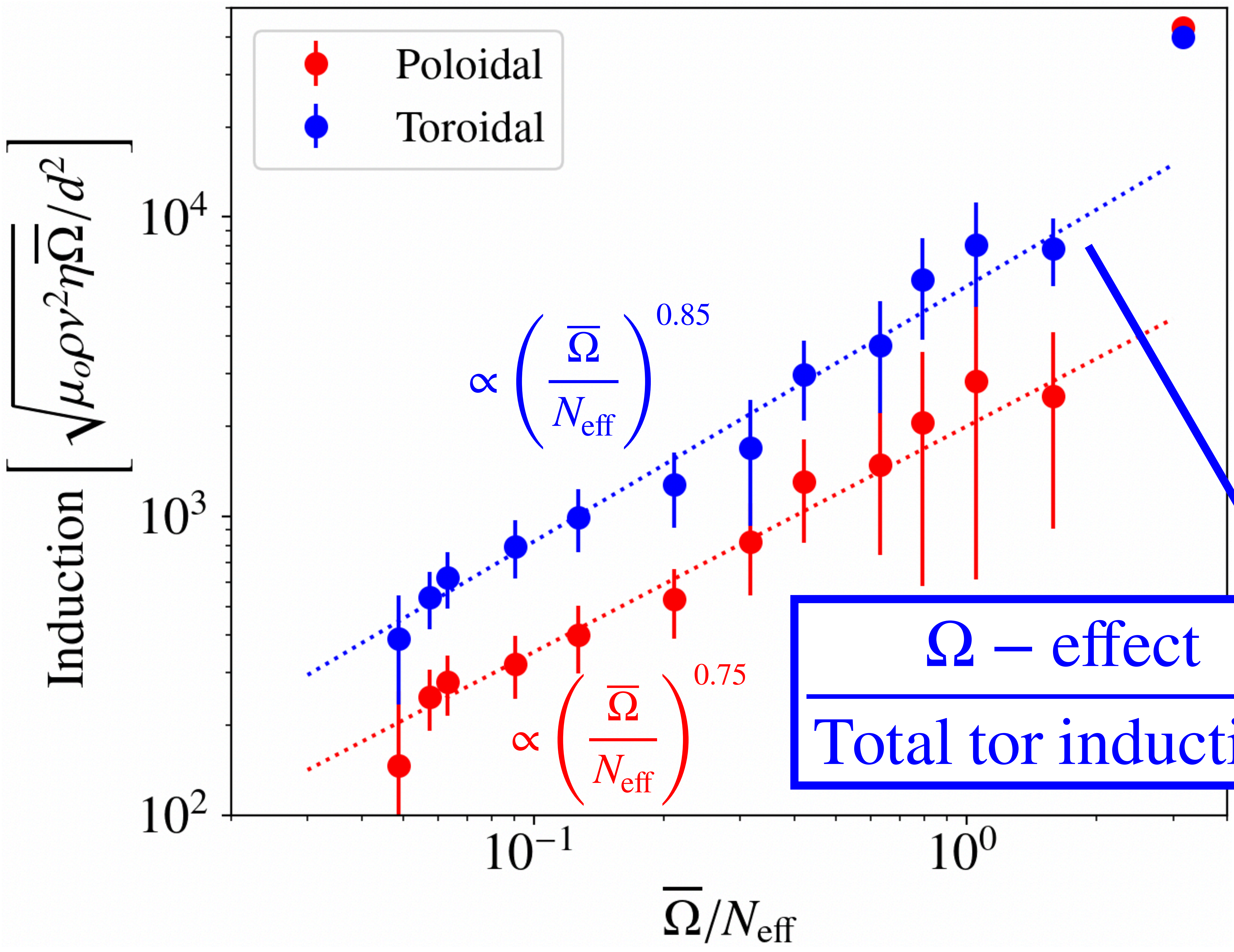




$\Longrightarrow$ Slightly less efficient AM transport than Fuller+19,
but much lower $q_{\min}$ in red giants

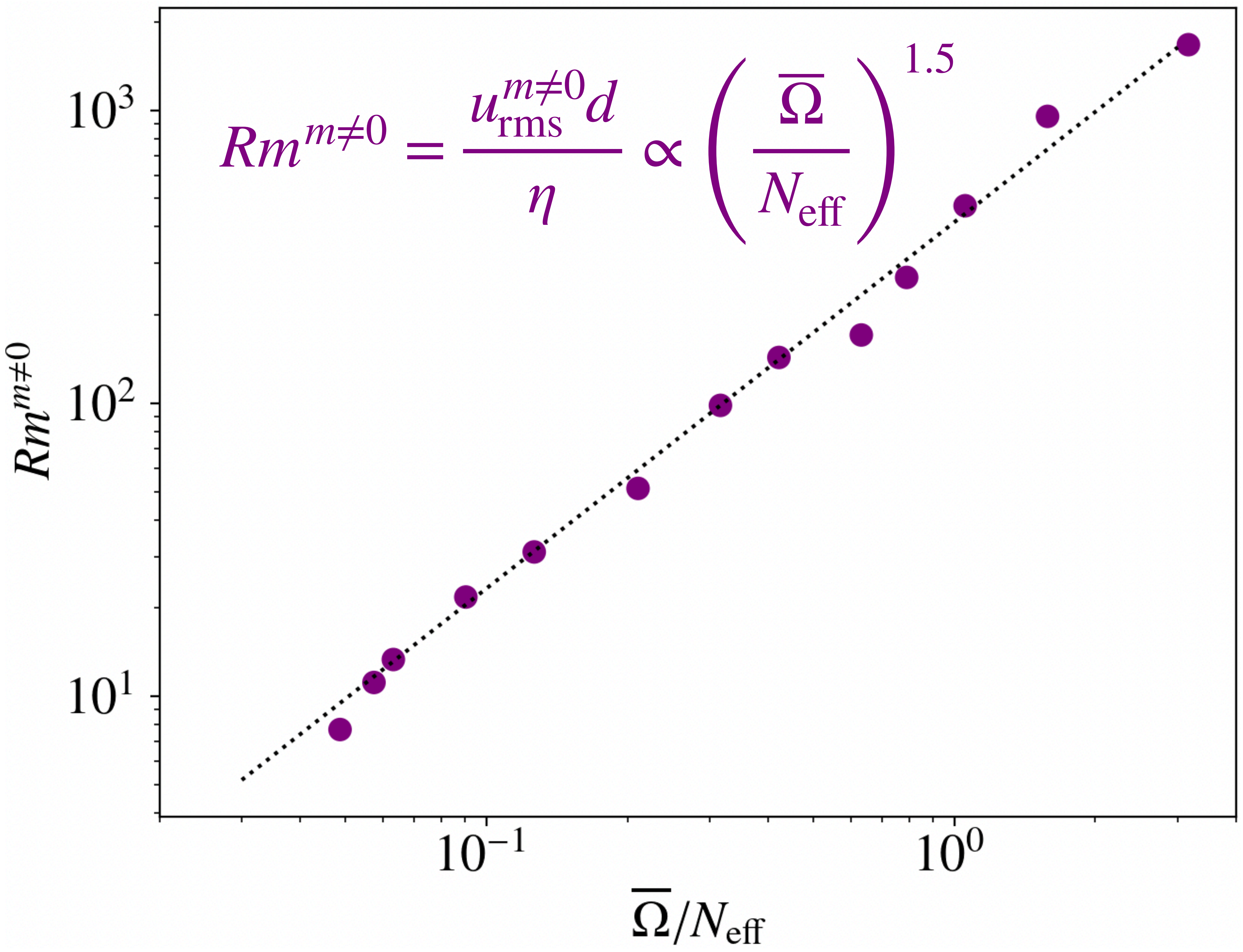

10^1
10^0
E_M/E_K
10^{-1}
10^0
$\overline{\Omega}/N_{\text{eff}}$

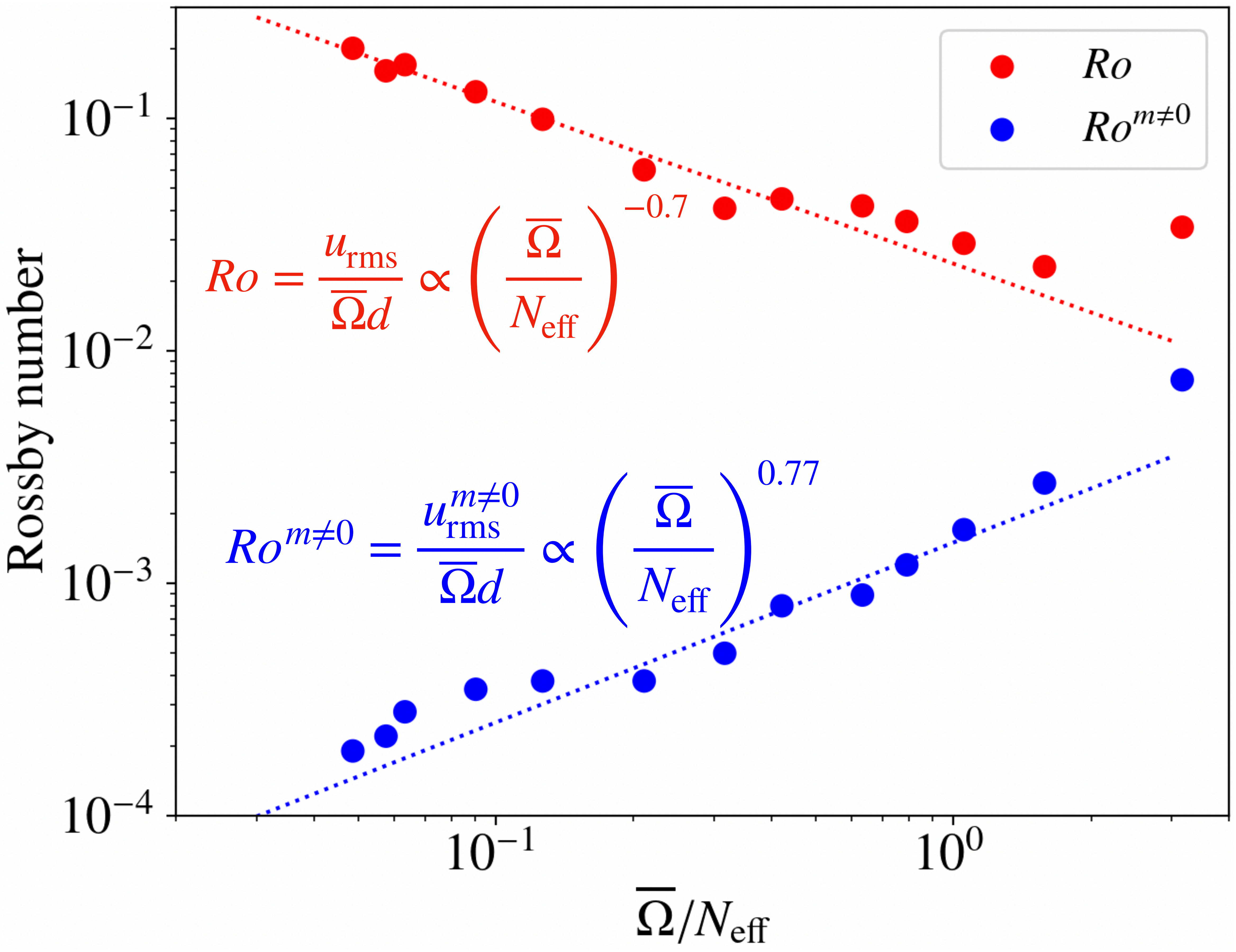

10^1
10^0
$E_M^{m\neq0}/E_K^{m\neq0}$
10^{-1}
10^0
$\overline{\Omega}/N_{\text{eff}}$



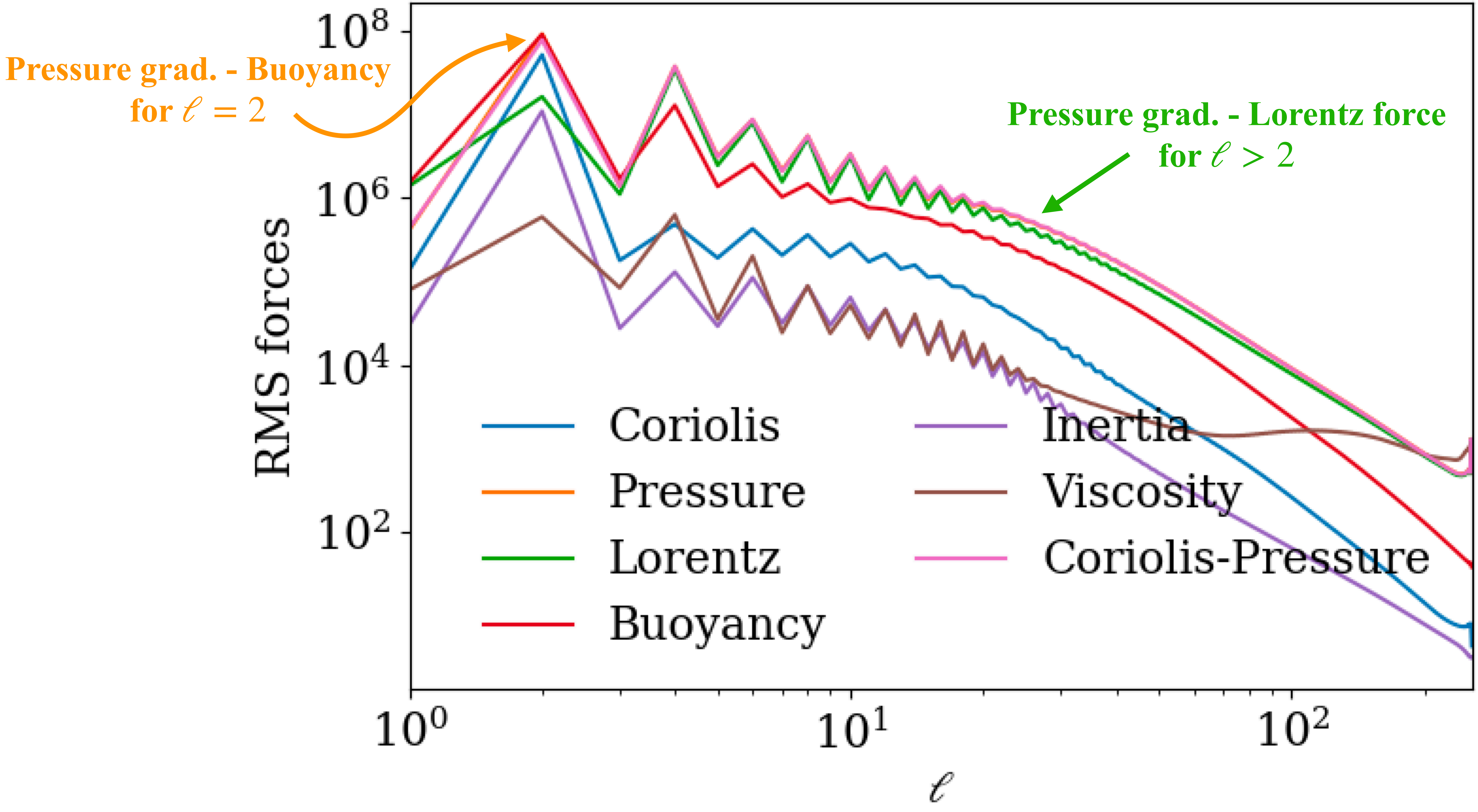
$$\frac{\Omega - \text{effect}}{\text{Total tor induction}} = \frac{\nabla \times (u_\phi \mathbf{e}_\phi \times \mathbf{B})}{\nabla \times (\mathbf{u} \times \mathbf{B})} \sim 0.6$$

Critical Rossby and magnetic Reynolds numbers





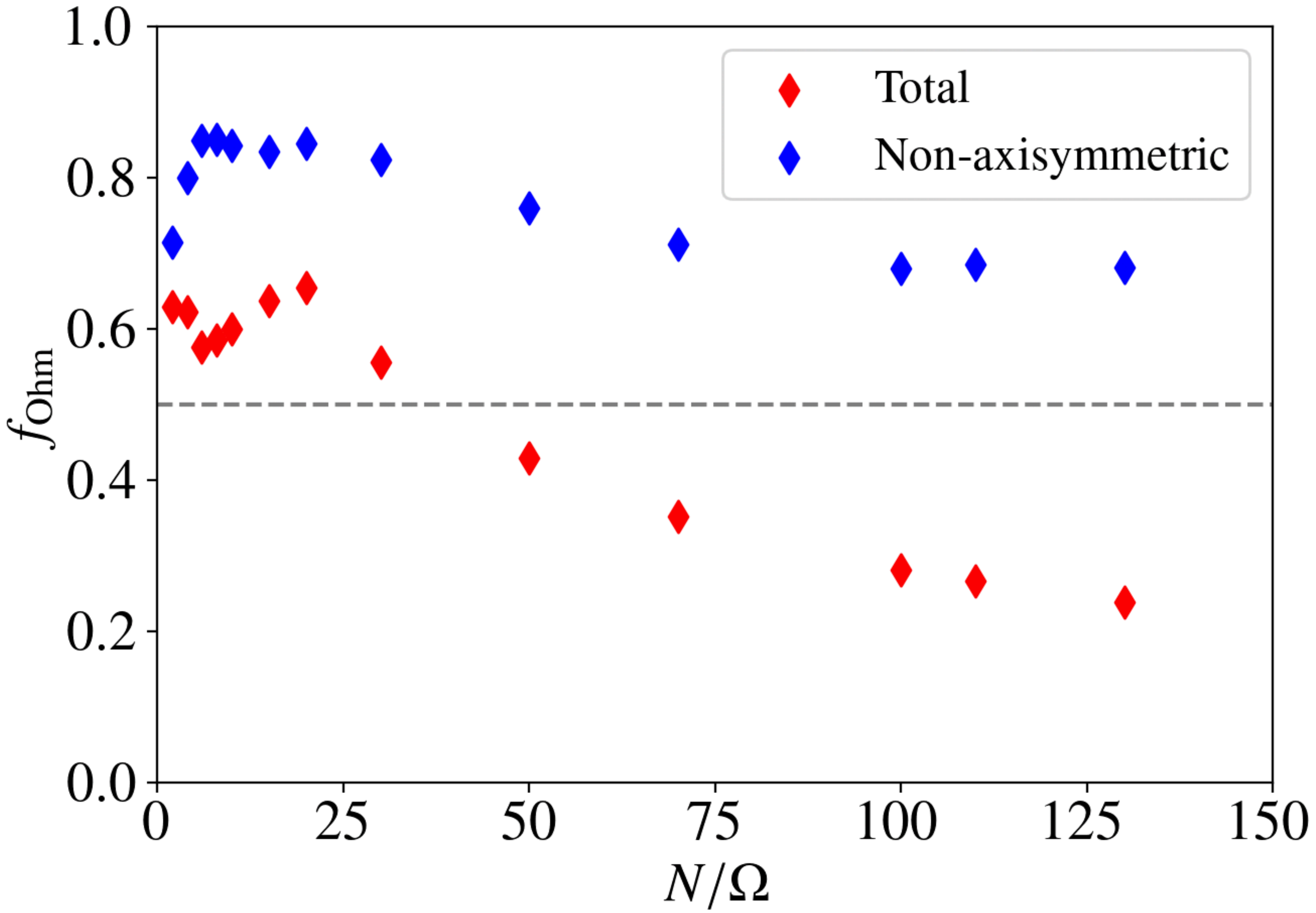
Force balance spectrum



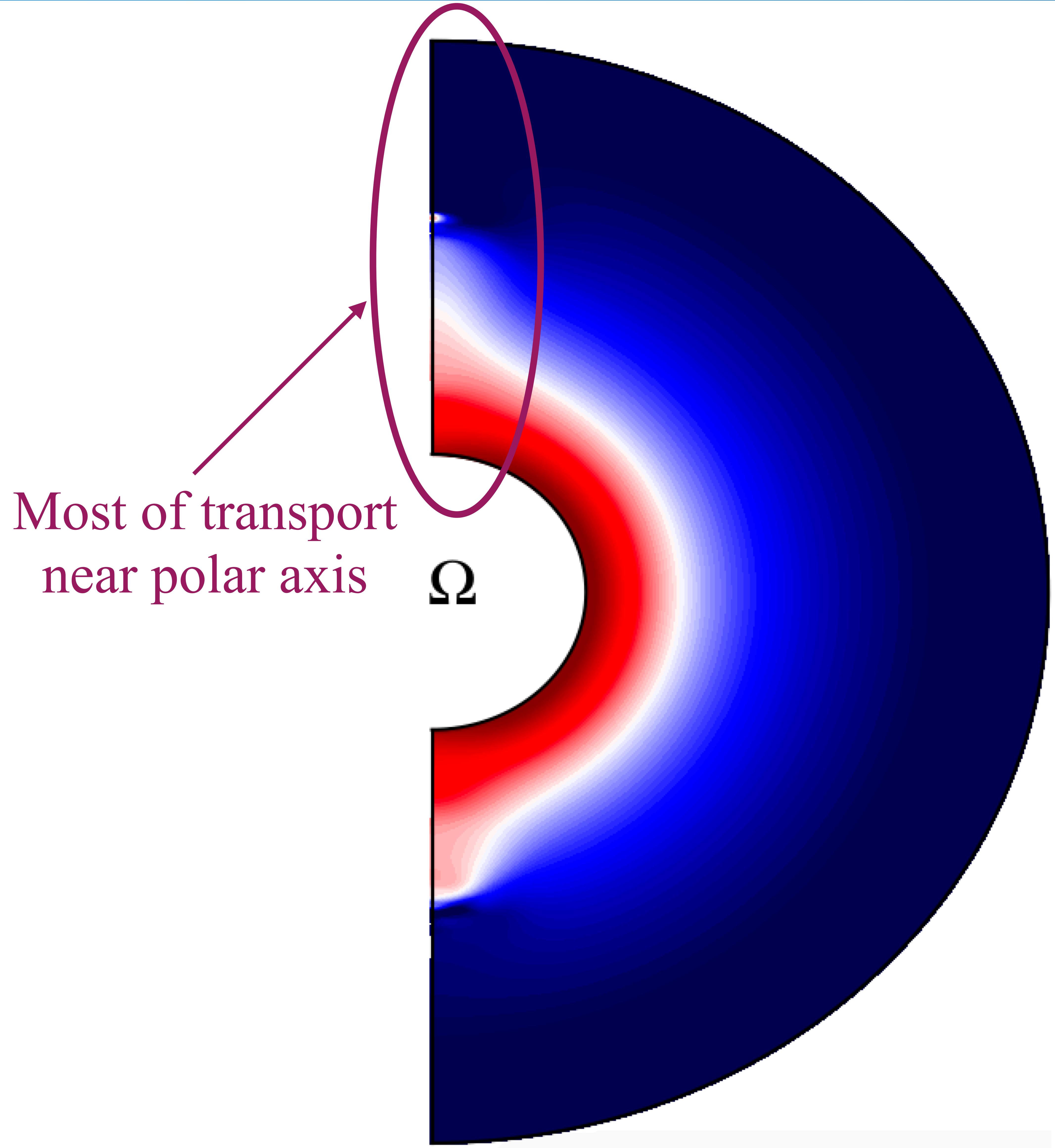
Dissipations

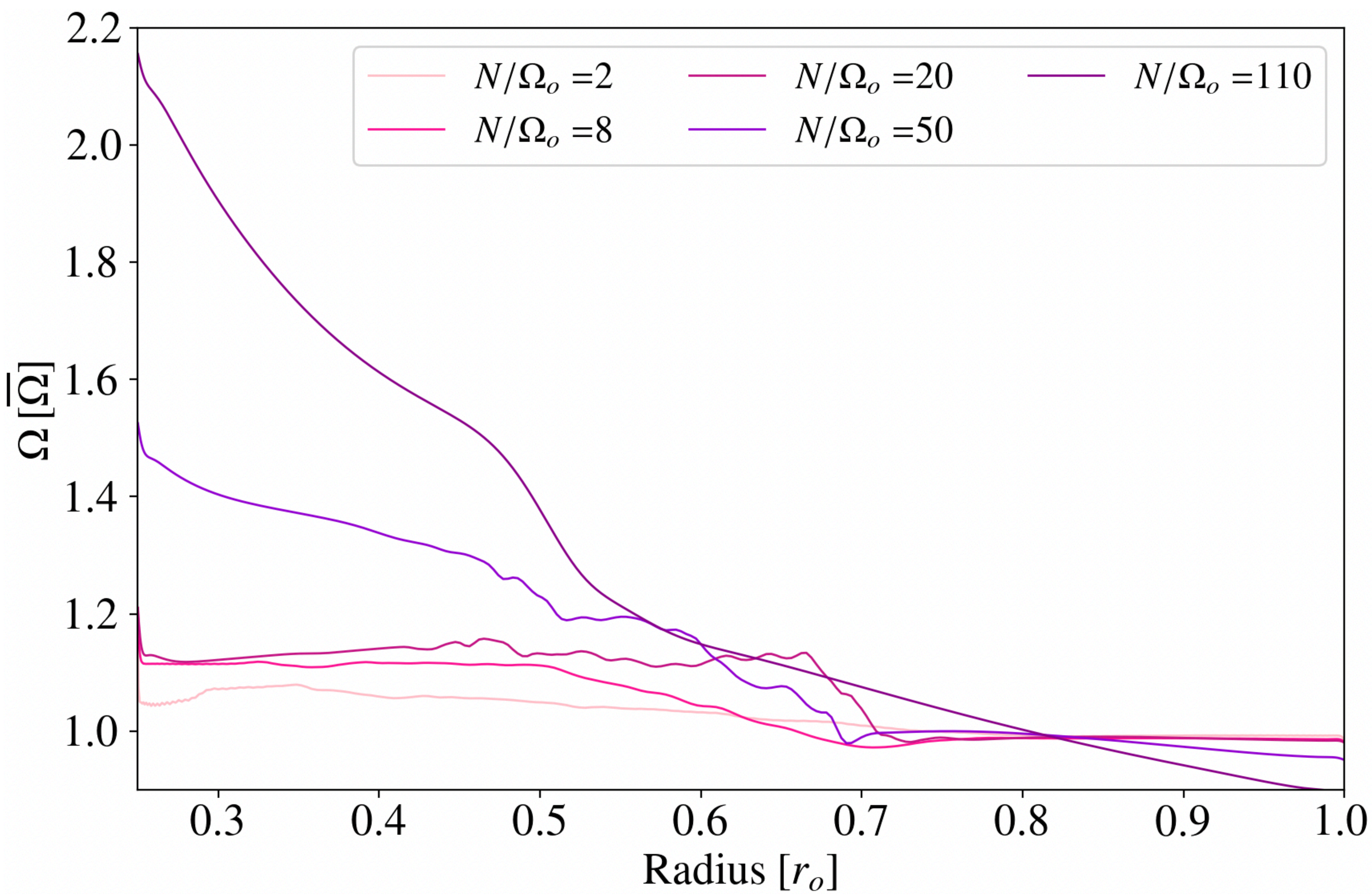
$f_{\mathrm{Ohm}} \equiv Q^{\eta}/(Q^{\nu} + Q^{\eta})$

with $Q^{\eta} \propto (\nabla \times \mathbf{B})^2$
and $Q^{\nu} \propto (\nabla \times \mathbf{v})^2$



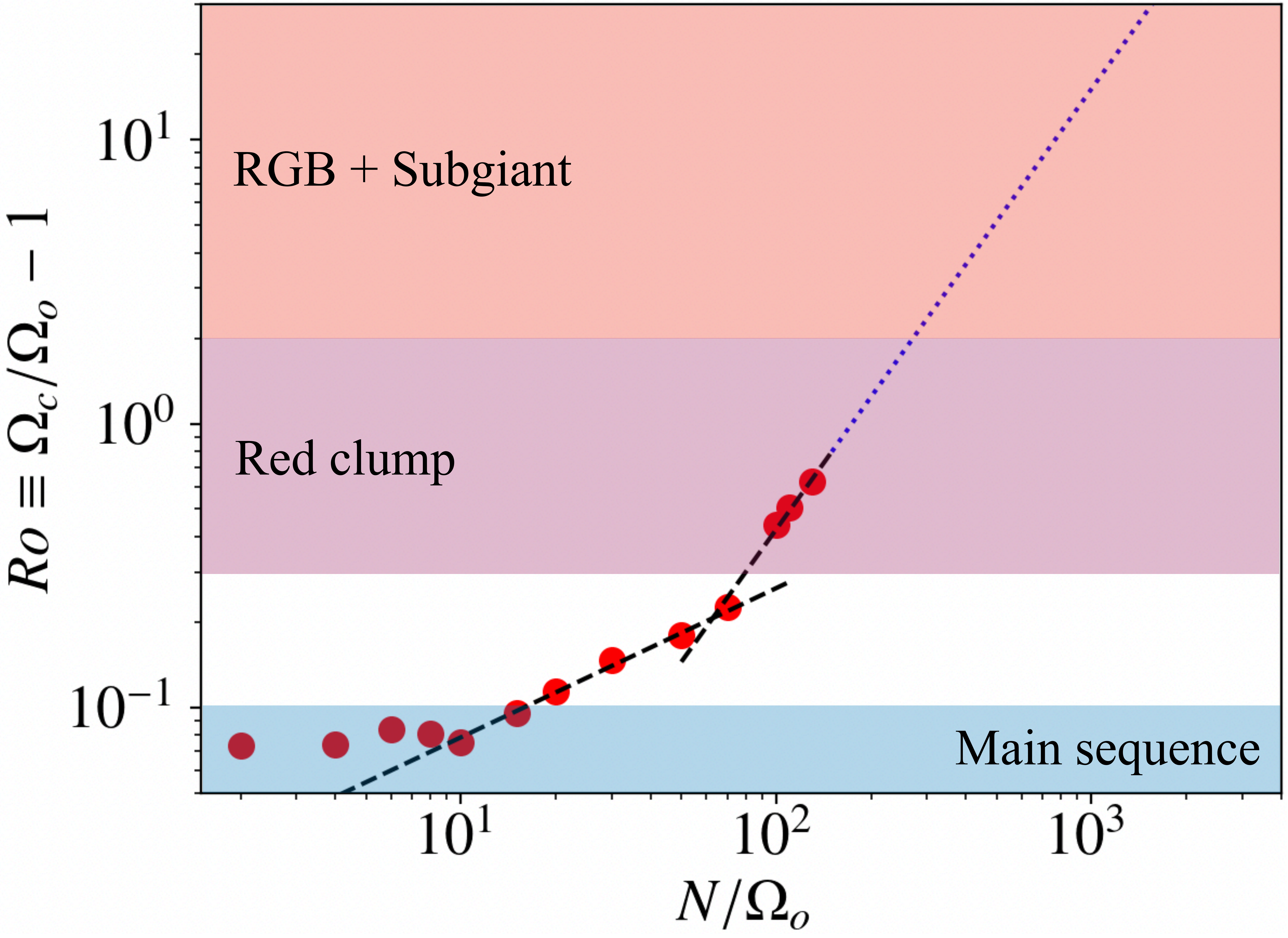
Local vs. global analysis



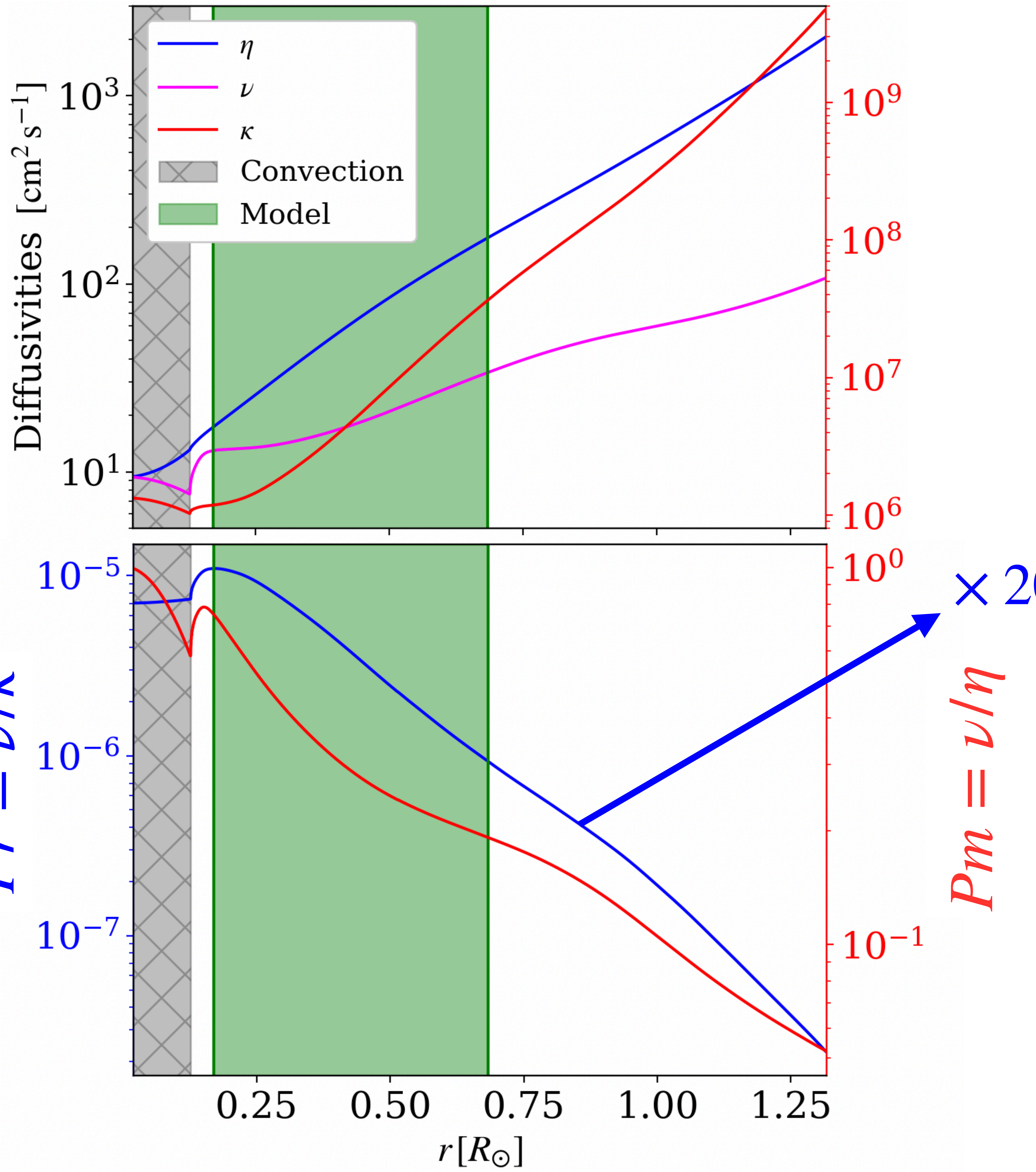


3D nature of the dynamo is important and complicates the application to stellar evolution models

Ratio core/env. rotation rate



Implementing realistic diffusivity radial dependence



$\times$ 2000 in the simulation
to be resolved

More vigorous dynamo, but more difficult to resolve radially…