Multifluid module in Dyablo & Shamrock

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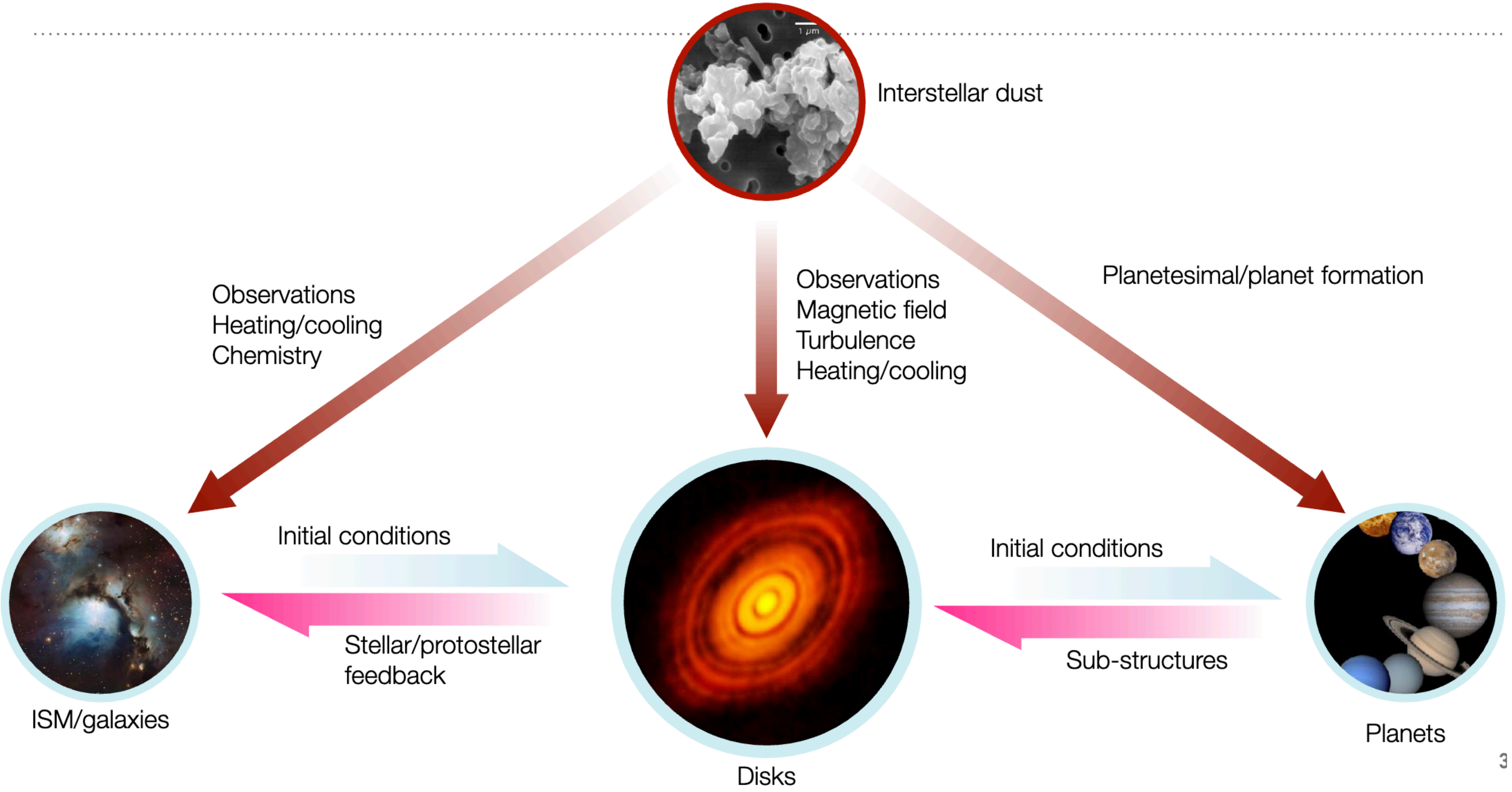
Dust is everywhere

- Solid bodies made of silicates, carbon
- Only **1%** of the mass budget in the diffuse ISM (Mathis et al. 1977; Weingartner & Draine 2001)



Credit : Ugo Lebreuilly

Epstein drag :
(*Relevant for most astrophysical object*)

$$F_{\text{drag}} \propto \frac{1}{T_s} \, , \qquad T_\text{s} \equiv \frac{\rho_{\text{grain}} s_{\text{grain}}}{\rho_\text{g} c_\text{s}} \sqrt{\frac{\pi\gamma}{8}}$$

Small grain / small stopping time $\Longrightarrow$ Strong coupling

Large grain / large stopping time $\Longrightarrow$ Weak coupling

Equations of evolution : 1 dust species

$$\frac{\partial \rho_{\mathrm{g}}}{\partial t} + \nabla \cdot \left(\rho_{\mathrm{g}} \mathbf{v}_{\mathrm{g}}\right) = 0,$$

$$\frac{\partial \rho_{\mathrm{d},i}}{\partial t} + \nabla \cdot \left(\rho_{\mathrm{d},i} \mathbf{v}_{\mathrm{d},i}\right) = 0,$$

$$\frac{\partial \left(\rho_{\mathrm{g}} \mathbf{v}_{\mathrm{g}}\right)}{\partial t} + \nabla \cdot \left(\rho_{\mathrm{g}} \mathbf{v}_{\mathrm{g}} \otimes \mathbf{v}_{\mathrm{g}} + P_{\mathrm{g}} \mathbf{I}\right) - \rho_{\mathrm{g}} \mathbf{f}_{\mathrm{g}} = \rho_{\mathrm{d}} \frac{\mathbf{v}_{\mathrm{d}} - \mathbf{v}_{\mathrm{g}}}{T_s},$$

$$\frac{\partial \left(\rho_{\mathrm{d}} \mathbf{v}_{\mathrm{d}}\right)}{\partial t} + \nabla \cdot \left(\rho_{\mathrm{d}} \mathbf{v}_{\mathrm{d}} \otimes \mathbf{v}_{d}\right) - \rho_{d} \mathbf{f}_{\mathrm{d}} = \rho_{\mathrm{d}} \frac{\mathbf{v}_{\mathrm{g}} - \mathbf{v}_{\mathrm{d}}}{T_{\mathrm{s}}},$$

pressureless dust fluid

$$\frac{\partial E_{\mathrm{g}}}{\partial t} + \nabla \cdot \left[\left(E_{\mathrm{g}} + P_{\mathrm{g}}\right) \mathbf{v}_{\mathrm{g}}\right] - \rho_{\mathrm{g}} \mathbf{f}_{\mathrm{g}} \cdot \mathbf{v}_{\mathrm{g}} = \rho_{\mathrm{d}} \frac{\mathbf{v}_{\mathrm{d}} - \mathbf{v}_{\mathrm{g}}}{T_s} \cdot \left((1 - w)\, \mathbf{v}_{\mathrm{g}} + w \rho_{\mathrm{d}}\right)$$

Equations of evolution : n dust species

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot F(\mathbf{U}) + \mathbf{f}_{\mathrm{src,ext}} = \mathbf{f}_{\mathrm{drag}}$$

$$\textbf{with } \mathbf{U} \equiv \begin{bmatrix} \rho_{\mathrm{g}} \\ \rho_{\mathrm{g}}\mathbf{v}_{\mathrm{g}} \\ E_{\mathrm{g}} \\ \rho_{\mathrm{d,1}} \\ \rho_{\mathrm{d,1}}\mathbf{v}_{\mathrm{d,1}} \\ \vdots \\ \rho_{\mathrm{d,n}} \\ \rho_{\mathrm{d,n}}\mathbf{v}_{\mathrm{d,n}} \end{bmatrix} F(\mathbf{U}) \equiv \begin{bmatrix} \rho_{\mathrm{g}}\mathbf{v}_{\mathrm{g}} \\ \rho_{\mathrm{g}}\mathbf{v}_{\mathrm{g}} \otimes \mathbf{v}_{\mathrm{g}} + P_{\mathrm{g}}\mathbf{I} \\ (E_{\mathrm{g}} + P_{\mathrm{g}})\mathbf{v}_{\mathrm{g}} \\ \rho_{\mathrm{d,1}}\mathbf{v}_{\mathrm{d,1}} \\ \rho_{\mathrm{d,1}}\mathbf{v}_{\mathrm{d,1}} \otimes \mathbf{v}_{\mathrm{d,1}} \\ \vdots \\ \rho_{\mathrm{d},n}\mathbf{v}_{\mathrm{d},n} \\ \rho_{\mathrm{d},n}\mathbf{v}_{\mathrm{d},n} \otimes \mathbf{v}_{\mathrm{d},n} \end{bmatrix} \quad \mathbf{f}_{\mathrm{src,ext}} \equiv \begin{bmatrix} 0 \\ \rho_{\mathrm{g}}\mathbf{f}_{\mathrm{g}} \\ \rho_{\mathrm{g}}\mathbf{f}_{\mathrm{g}} \cdot \mathbf{v}_{\mathrm{g}} \\ 0 \\ \rho_{\mathrm{d,1}}\mathbf{f}_{\mathrm{d,1}} \\ \vdots \\ 0 \\ \rho_{\mathrm{d},i}\mathbf{f}_{\mathrm{d},n} \end{bmatrix} \quad \mathbf{f}_{\mathrm{drag}} \equiv \begin{bmatrix} 0 \\ \sum_{i=1}^{N} \rho_{\mathrm{d},i} \frac{\mathbf{v}_{\mathrm{d,i}} - \mathbf{v}_{\mathrm{g}}}{T_{\mathrm{s},i}} \\ \sum_{i=1}^{N} \rho_{\mathrm{d},i} \frac{\mathbf{v}_{\mathrm{d},i} - \mathbf{v}_{\mathrm{g}}}{T_{\mathrm{s},i}} \cdot \mathbf{v}_{\mathrm{g}} + w \sum_{i=1}^{N} \rho_{\mathrm{d},i} \frac{(\mathbf{v}_{\mathrm{d},i} - \mathbf{v}_{\mathrm{g}})^2}{T_{\mathrm{s},i}} \\ 0 \\ \rho_{\mathrm{d,1}} \frac{\mathbf{v}_{\mathrm{g}} - \mathbf{v}_{\mathrm{d,1}}}{T_{\mathrm{s,1}}} \\ \vdots \\ 0 \\ \rho_{\mathrm{d},n} \frac{\mathbf{v}_{\mathrm{g}} - \mathbf{v}_{\mathrm{d},n}}{T_{\mathrm{s},n}} \end{bmatrix}$$

Numerical scheme

Full time step

0	1	2	…	$i-1$	i	$i+1$	…	N − 3	N − 2	N − 1
$\mathbf{U}_0^n$					$\mathbf{U}_i^n$					$\mathbf{U}_{N-1}^n$

Transport Step

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot F(\mathbf{U}) + \mathbf{f}_{\mathrm{src,ext}} = 0$$

0	1	2	…	$i-1$	i	$i+1$	…	N − 3	N − 2	N − 1
$\tilde{\mathbf{U}}_0^n$					$\tilde{\mathbf{U}}_i^n$					$\tilde{\mathbf{U}}_{N-1}^n$

Drag Step

$$\frac{\mathrm{d}\tilde{\mathbf{U}}_0^n}{\mathrm{dt}} = \Omega \tilde{\mathbf{U}}_0^n \qquad \frac{\mathrm{d}\tilde{\mathbf{U}}_i^n}{\mathrm{dt}} = \Omega \tilde{\mathbf{U}}_i^n \qquad \frac{\mathrm{d}\tilde{\mathbf{U}}_{N-1}^n}{\mathrm{dt}} = \Omega \tilde{\mathbf{U}}_{N-1}^n$$

0	1	2	…	$i-1$	i	$i+1$	…	N − 3	N − 2	N − 1
$\mathbf{U}_0^{n+1}$					$\mathbf{U}_i^{n+1}$					$\mathbf{U}_{N-1}^{n+1}$

Numerical scheme : Drag step

$$\frac{\mathrm{d}\tilde{\mathbf{U}}}{\mathrm{dt}} = \boldsymbol{\Omega}\tilde{\mathbf{U}} \,, \qquad \text{where} \qquad \boldsymbol{\Omega} \equiv \begin{bmatrix} -\sum_{i=1}^{N} \epsilon_i \alpha_i & \alpha_1 & \alpha_2 & \dots & \alpha_N \\ \epsilon_1 \alpha_1 & -\alpha_1 & 0 & \dots & 0 \\ \epsilon_2 \alpha_2 & 0 & -\alpha_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \epsilon_N \alpha_N & 0 & 0 & \dots & -\alpha_N \end{bmatrix} \qquad \text{and} \qquad \begin{aligned} \epsilon_i &\equiv \frac{\rho_{\mathrm{d},i}}{\rho_{\mathrm{g}}} \text{ (dust-to-gas-ratio)} \\ \alpha_i &\equiv \frac{1}{T_{\mathrm{s},i}} \text{ (collision rate)} \end{aligned}$$

First-order Runge-Kutta implicit scheme in FARGO3D (Benitez-Llambert et al., 2019), RAMSES (Verrier et al., 2025)

Second-order Van Leer, Second-order Runge-Kutta and other schemes in ATHENA++ (Huang & Bai, 2022)

Second-order Diagonal implicit Runge Kutta scheme (Krapp et al., 2024)

$$\frac{d\tilde{\mathbf{U}}}{dt} = \Omega\tilde{\mathbf{U}} \qquad \Longrightarrow \qquad \tilde{\mathbf{U}}(t) = e^{t\Omega}\tilde{\mathbf{U}}\left(t = t_0\right)$$

High accurate scheme

General (sparse/dense)

Minimal cost

GPUs friendly

$$e^{A} = \left(e^{A/n}\right)^{n}, \quad n = 2^{s} \in \mathbf{N}.$$

$$\tilde{A} \simeq e^{2^{-s}A} \qquad \Rightarrow \qquad e^{A} \simeq \tilde{A}^{2^{s}}$$

Scaling step

Squaring step

Backward error bounded by the unit roundoff

Scaling factor s and Taylor polynomial order m **:**

Minimal cost (scaling step)

avoid overshooting (squaring step)

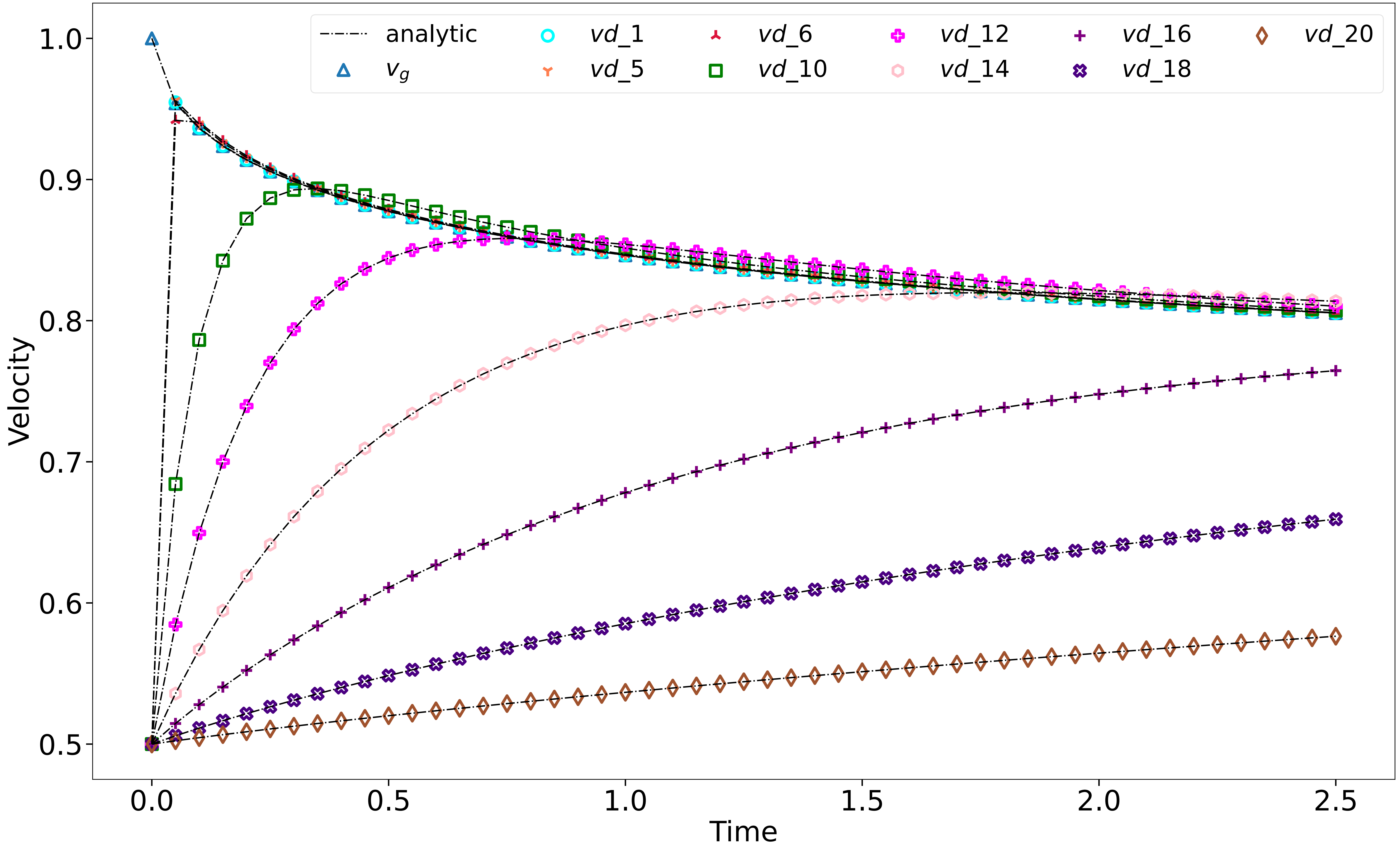
Dyablo

IRK1, IRK2, MDIRK, EXPO

Shamrock

IRK1, EXPO

$$\frac{d\tilde{\mathbf{U}}}{dt} = \Omega\tilde{\mathbf{U}}$$



Dusty collision : 20 dust species

Sewanou et al, 2025

Drag solver's validation (Two extreme regimes)

$$\frac{d\tilde{\mathbf{U}}}{dt} = \Omega\tilde{\mathbf{U}}$$

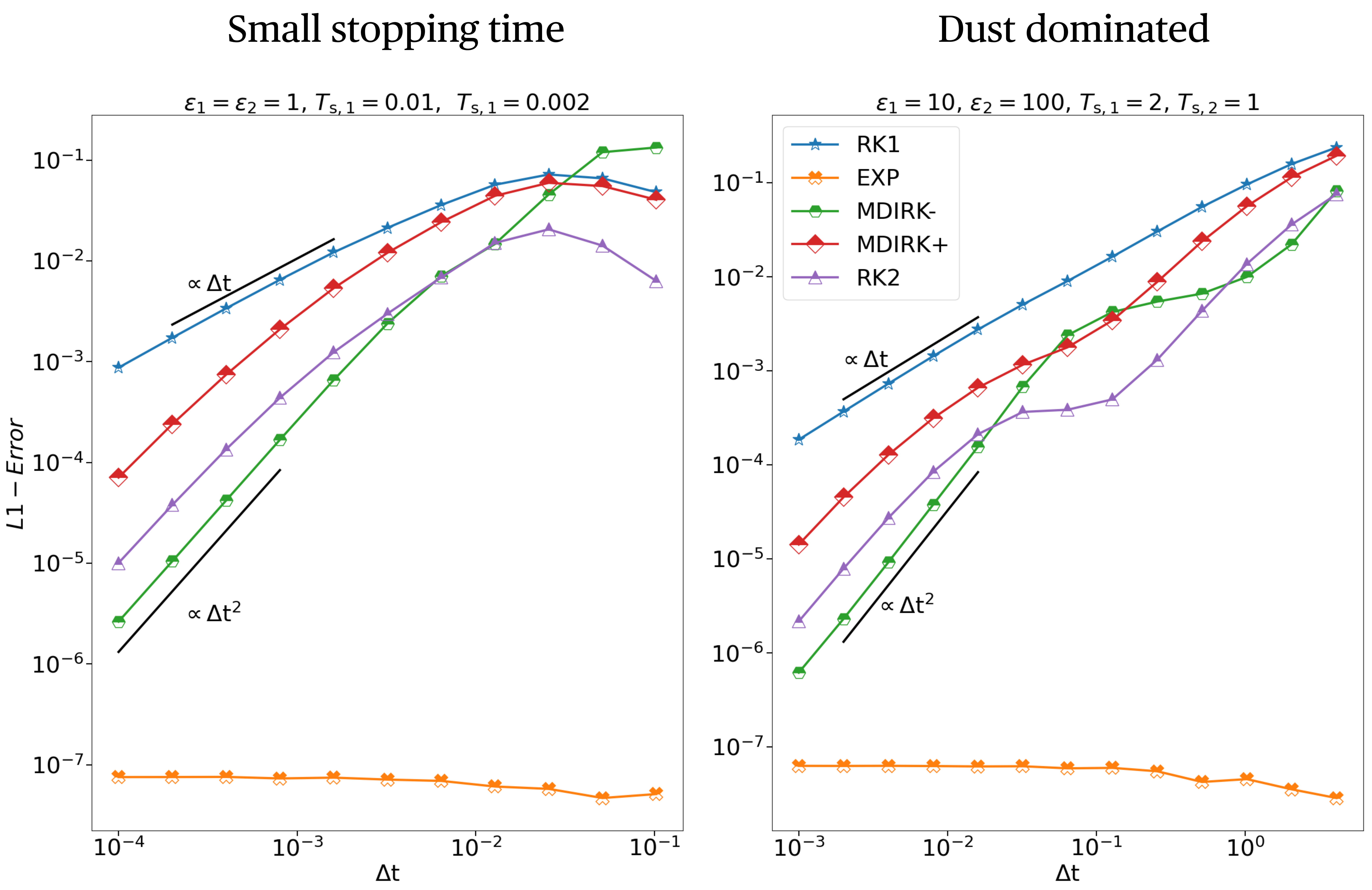
Error $\propto \Delta t$,

Time-to-solution $\propto 1/\Delta t$

10^4 (2 dust species)

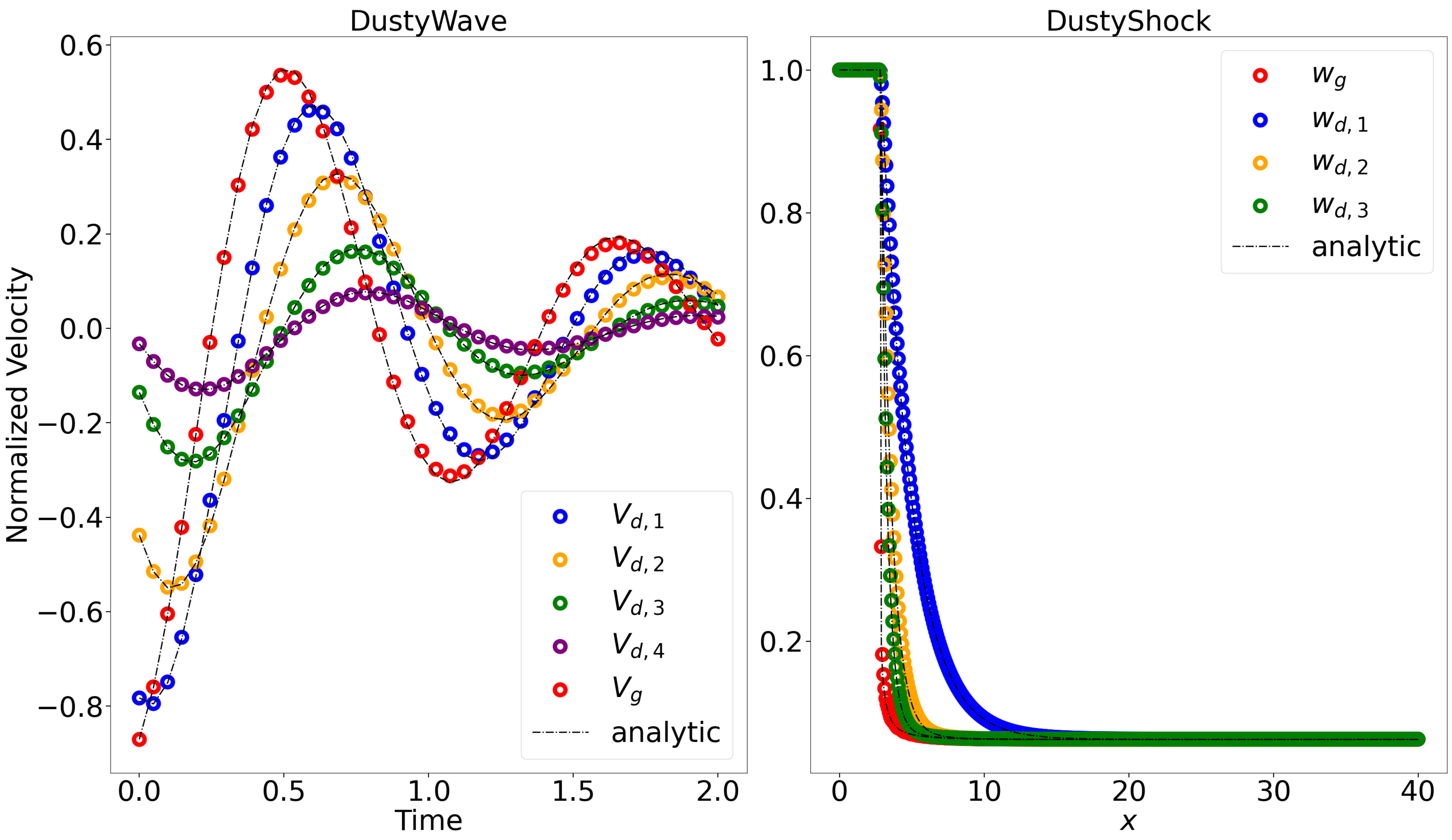
55 (20 dust species)

Sewanou et al, 2025



Coupling transport and drag

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot F(\mathbf{U}) + \mathbf{f}_{\text{src,ext}} = \mathbf{f}_{\text{drag}}$$



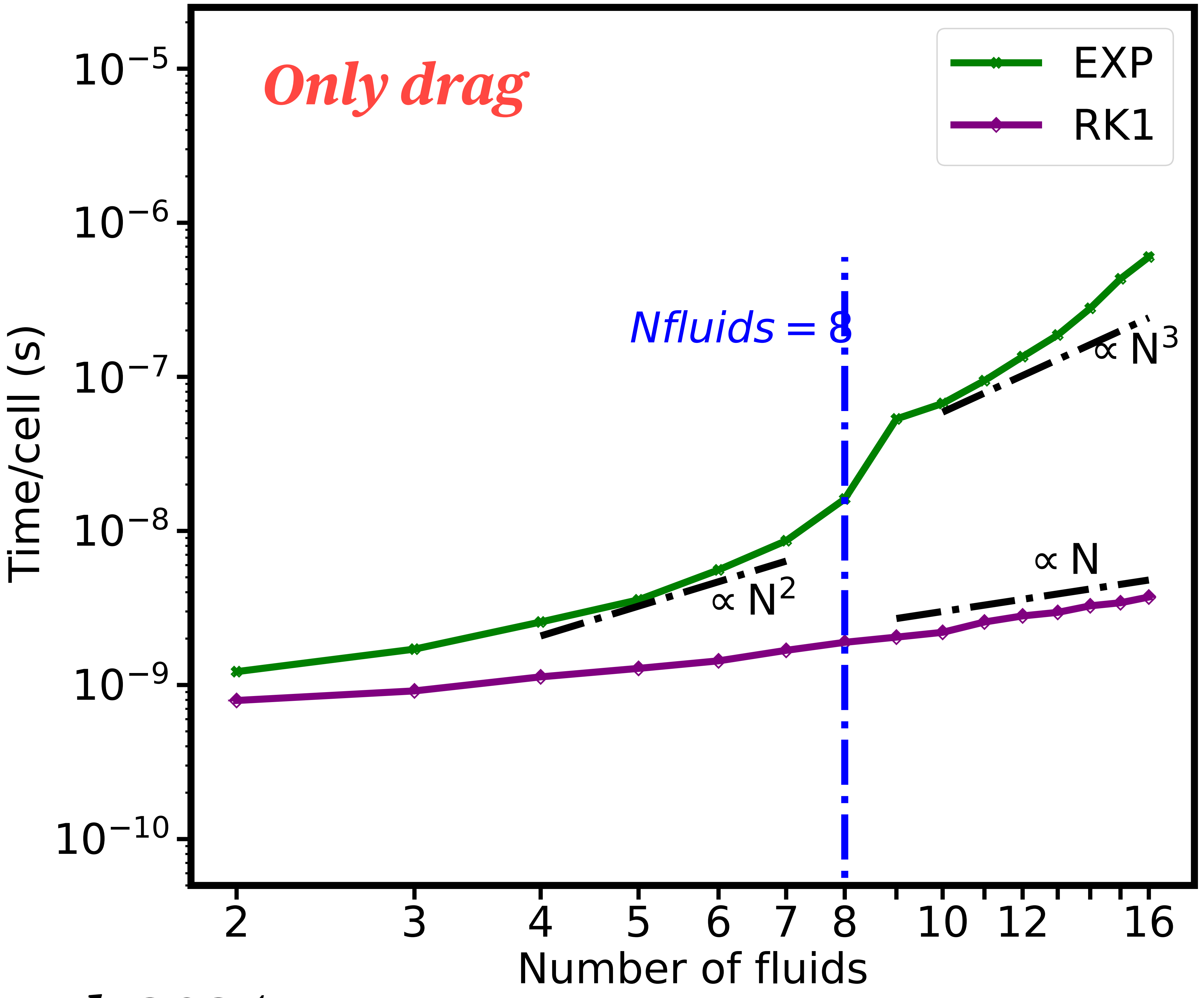
Sewanou et al, 2025

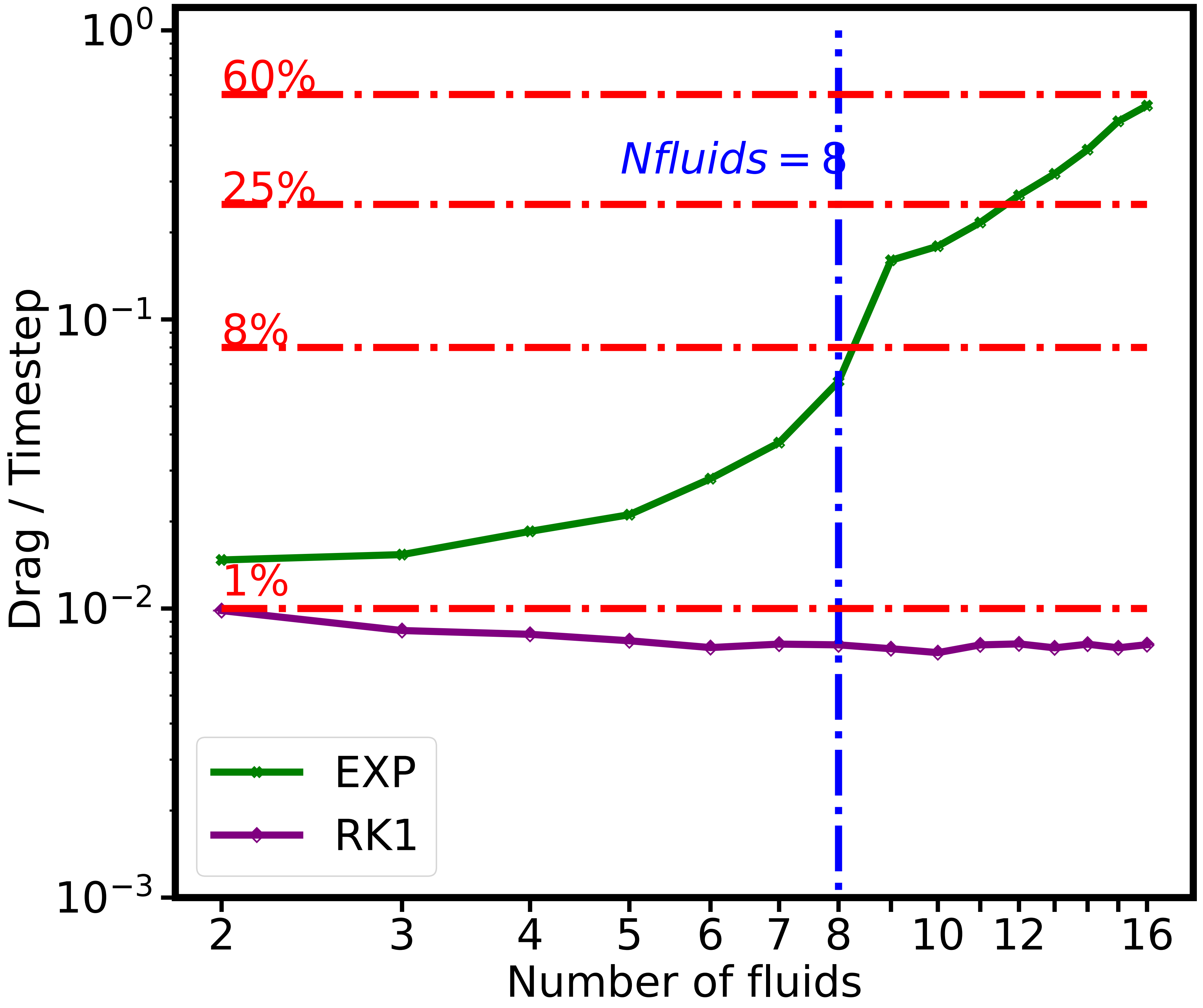
GPU A100

Nsight Compute + Nsight Systems

Dustywave test

64^3 cells





Sewanou et al, 2025

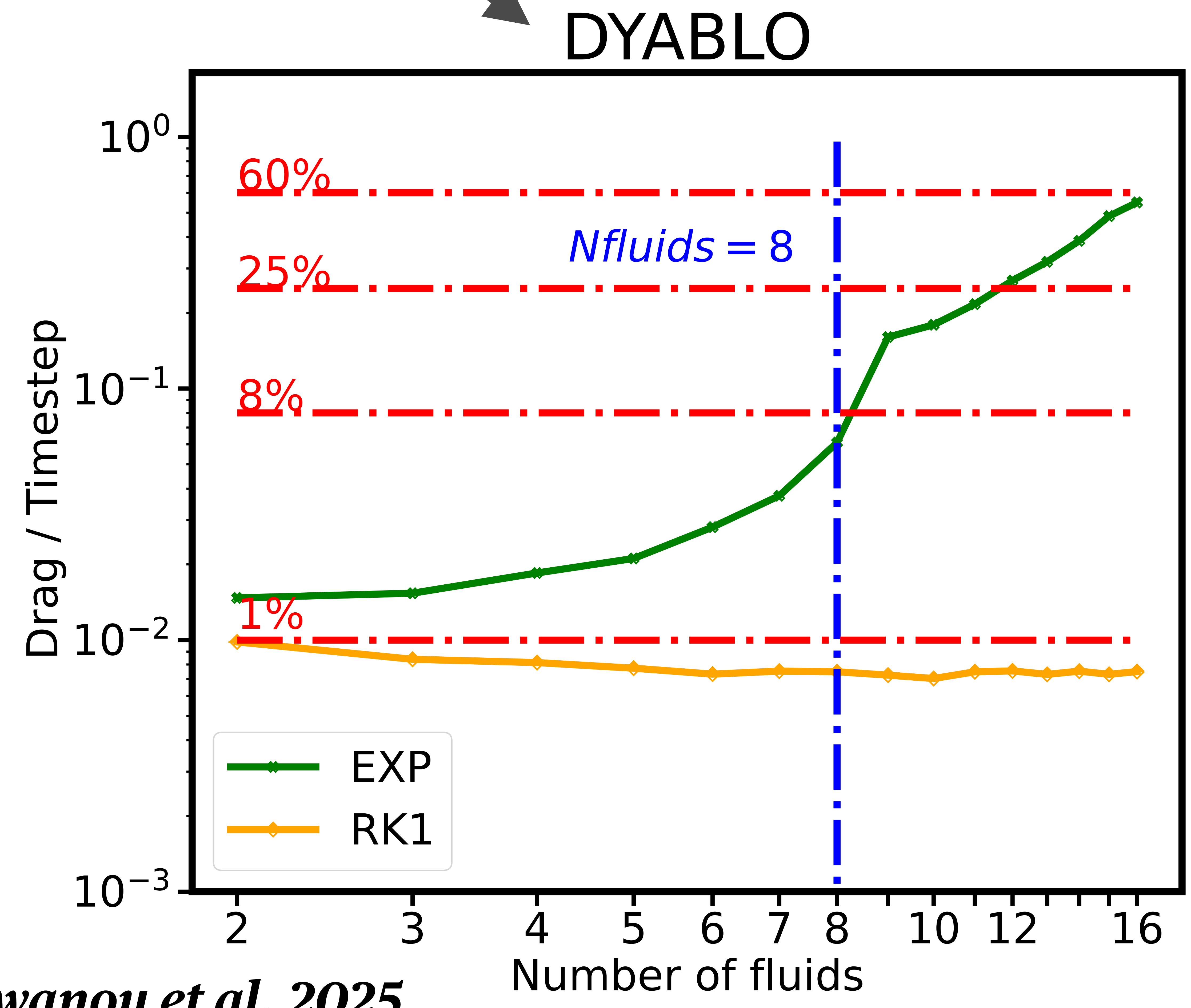
Performances on GPU (with Dyablo & Shamrock)

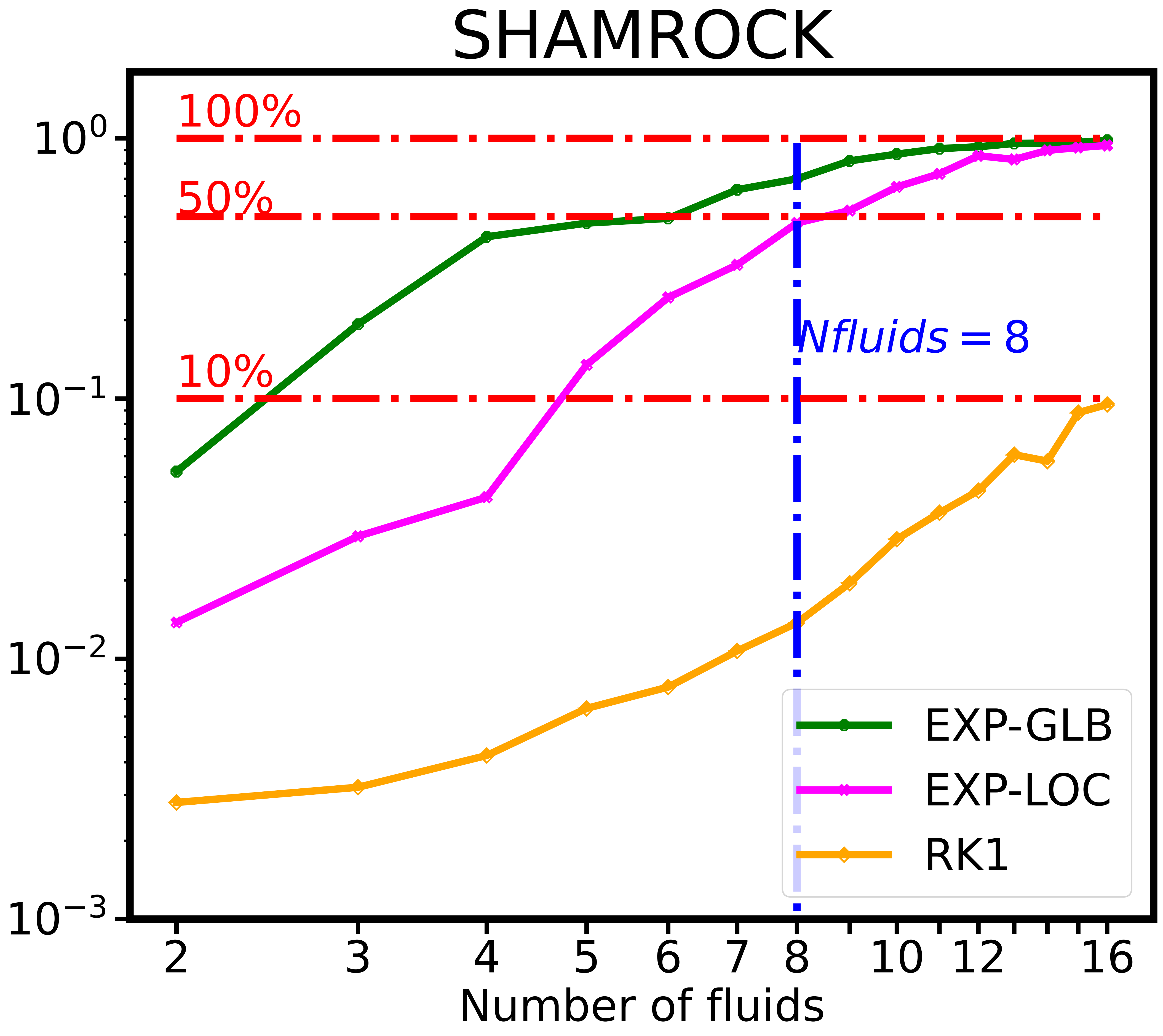
Static array

Dustywave test

64^3 cells

Dynamic array





Sewanou et al, 2025

Takeaways - Multifluids

High accurate drag solver on GPUs with Taylor based scaling and squaring matrix exponential computation

Time step independent

Implementation in Dyablo and Shamrock

Acceptable computational budget up to 16 dust species with current implementation

Application for other linear(ized) source terms (radiative transfer, chemistry)