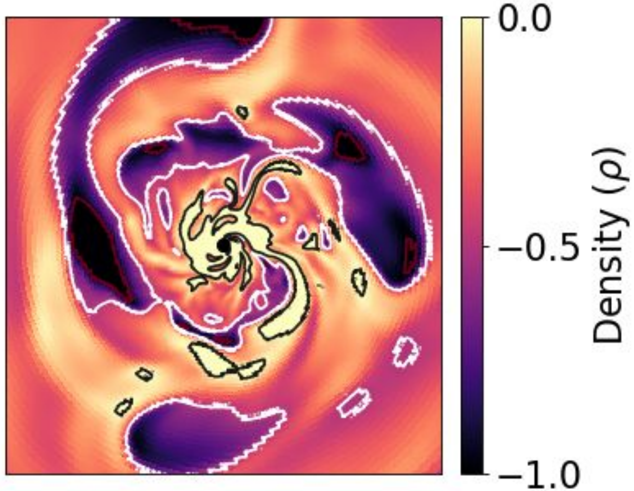
cuHARM: general relativistic radiation magneto-hydrodynamics in the era of exascale computing



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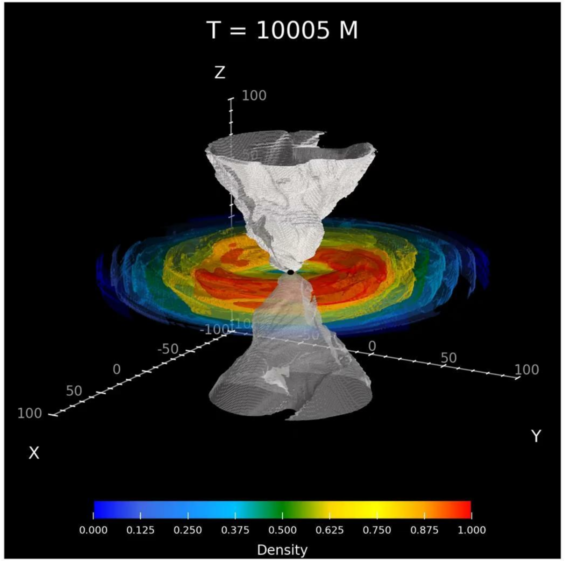

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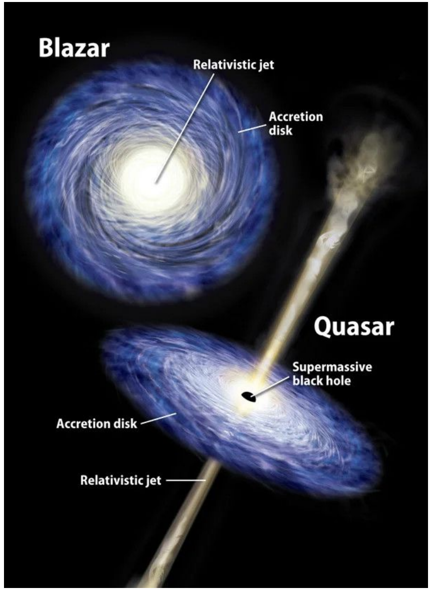
ASNUM, Grenoble, 16th of December 2025

Astrophysics is complicated



Source of energy: Gravitational / Rotational

(Magnetized/relativistic) jet / outflow

Kinetic / magnetic energy

Dissipation: Reconnection / shock / turbulence

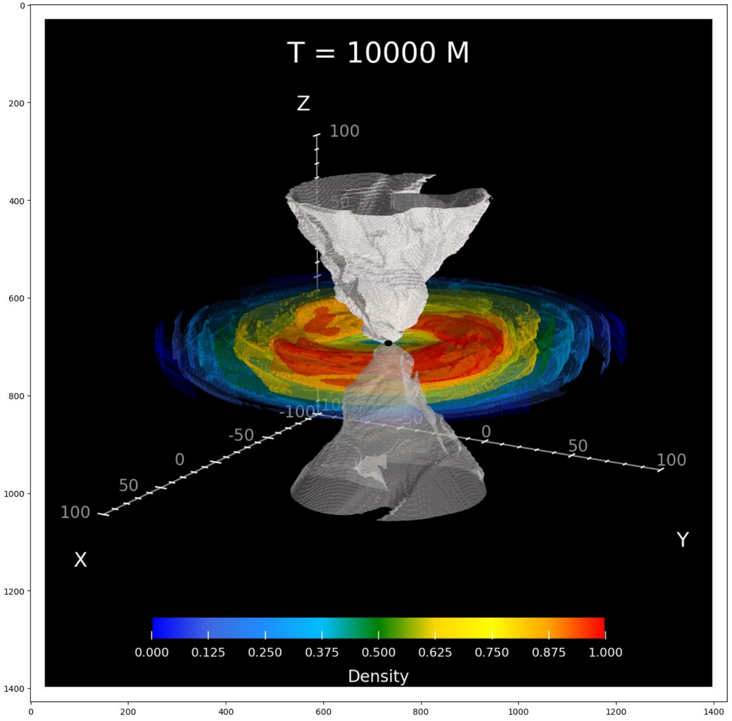
Particle heating / acceleration

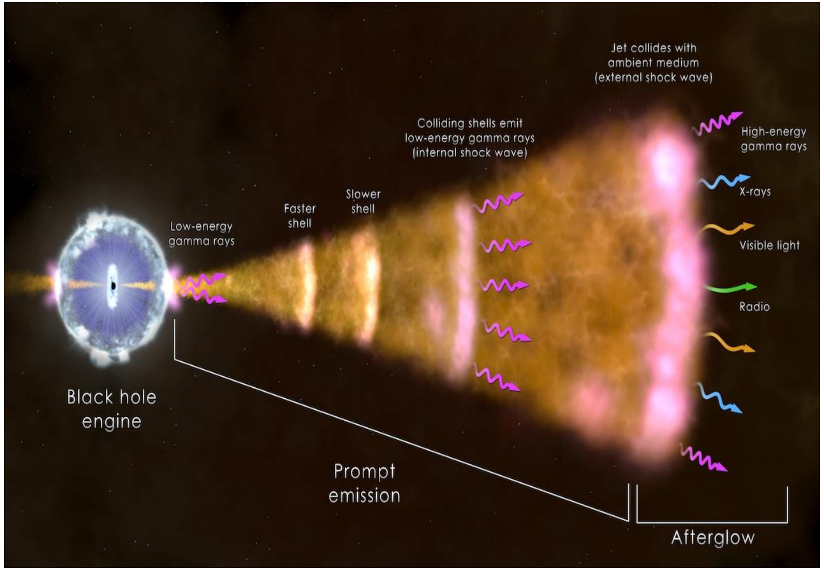
Electromagnetic signal
(light-curve, spectrum, polarization)

Gravitational waves, cosmic rays,
neutrinos

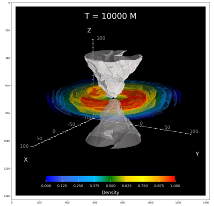
Introduction

Dynamics and Radiation





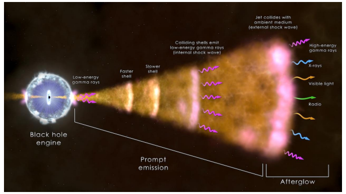
Introduction



Dynamics

- No emission
- Radiation usually neglected
- If not neglected: “simplified” treatment to only account for its dynamical contribution.

Radiation



Simplified dynamical model

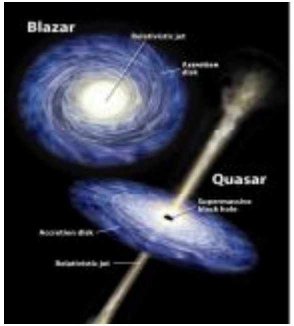
- “blob” model (blazar),
- infinitely thin shell (GRBs)....
- Self-similar motion (GRB afterglow)
- Post-processing ray tracing.

Yet .... dynamics and radiation are tightly linked:

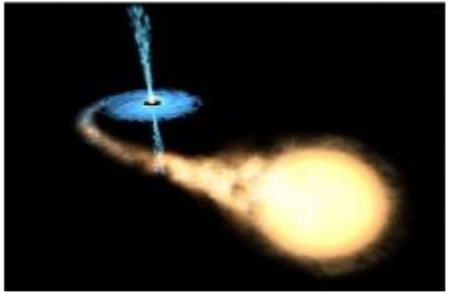
- Observables (light curve, spectrum) strongly depends on the dynamics:
  - Geometry, cooling state of the gas, velocity field ...
- The dynamics is modified by radiative output.

On the importance of radiation feedback

Eddington luminosity: $$L_{Edd} = \frac{4\pi G M m_p c}{\sigma_T} \approx 1.3\times 10^{38}\left(\frac{M}{M_{\odot}}\right) erg.s^{-1}$$



Active Galactic Nuclei



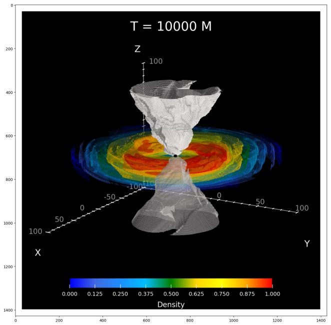
X-ray binaries

Gamma-ray bursts

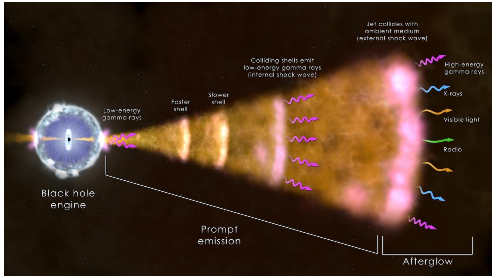
Other sources:

Ultra-luminous X-ray sources (ULXs), Tidal disruption events (TDEs), Supernovae

Introduction: how to bridge in a simulation radiation and dynamics?



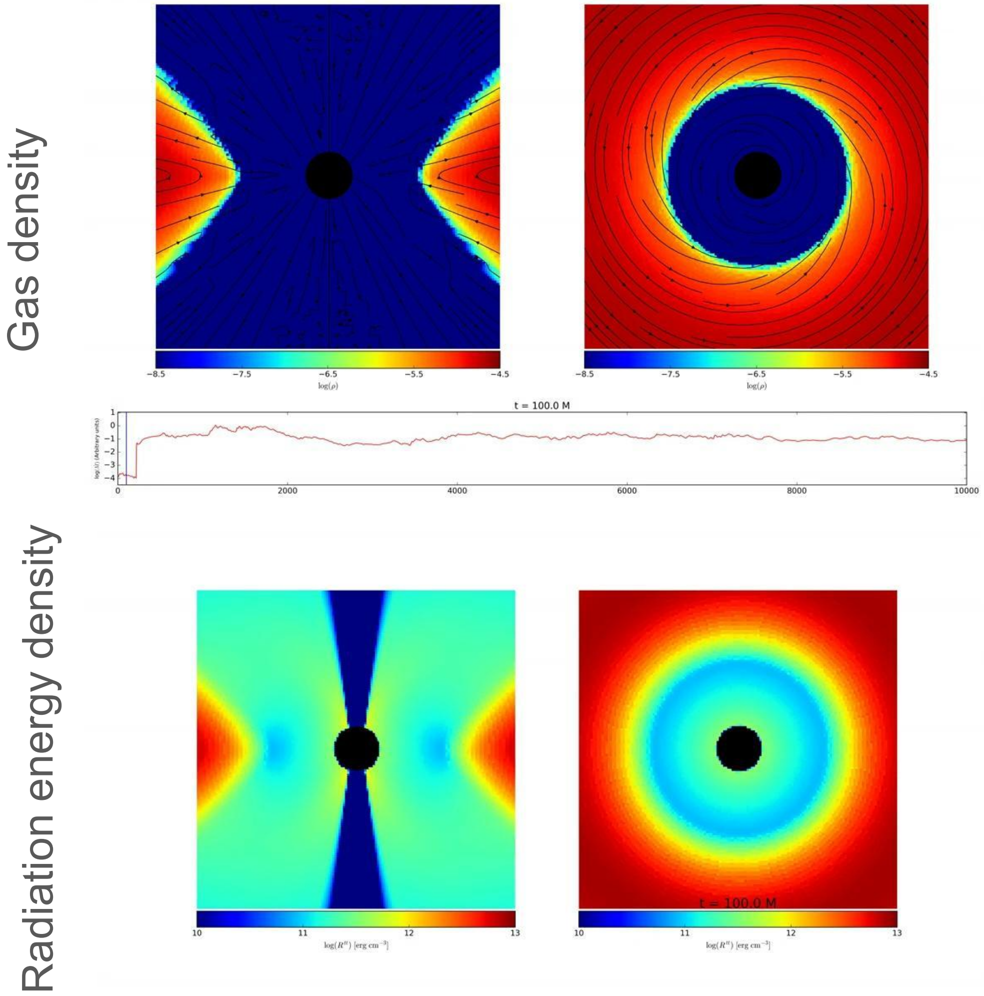
Dynamics ⟷ Radiation



Goals:

- Self-consistently calculate the radiative contribution to the dynamics:
  Cooling of the gas, radiation force, radiation anisotropy, radiation spectrum.
- Self-consistently integrate the dynamical effects on the produced radiation.

Bottom line: GR-R-MHD with angular resolution of the radiation field.



The dynamics: (GR) Magnetohydrodynamics (MHD)

Study the motion of magnetized plasma in the close vicinity of a black-hole

Study the motion of magnetized plasma in relativistic jets

We need a (GR)-MHD code

Conservation of gas density

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\rho u^\mu\right) = 0 \qquad (1)$$

Energy and momentum conservation

$$T^\mu{}_{\nu;\mu} = 0 \qquad (2)$$

Maxwell Equations

$$F_{\mu\nu,\lambda} + F_{\lambda\mu,\nu} + F_{\nu\lambda,\mu} = 0 \qquad (3)$$

Komissarov (1999, 2001), Gammie et al. (2003), Font (2008), and Rezzolla & Zanotti (2013)

The dynamics: cuHARM in a nutshell

We designed and built cuHARM.

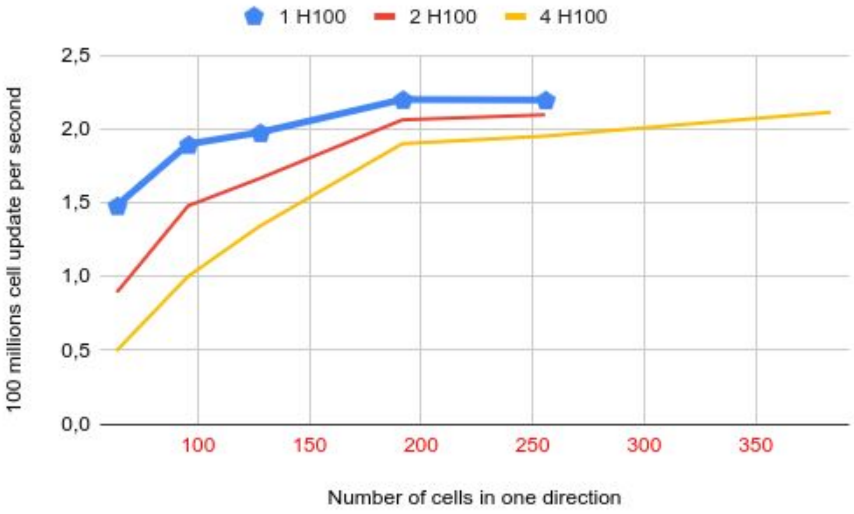
- ❖ **3D GR-MHD code:**
  - ➢ Finite volume
  - ➢ Flux-CT to preserve div B = 0
- ❖ Designed for multi-GPUs nodes: **CUDA-C / openMP / MPI**
- ❖ Thoroughly tested (e.g. comparison with Porth et al. 2019)
- ❖ **Highly optimized:**
  - ➢ >~10^8 cell updates per second on a A100
  - ➢ Two versions: highly optimized vs easily modifiable

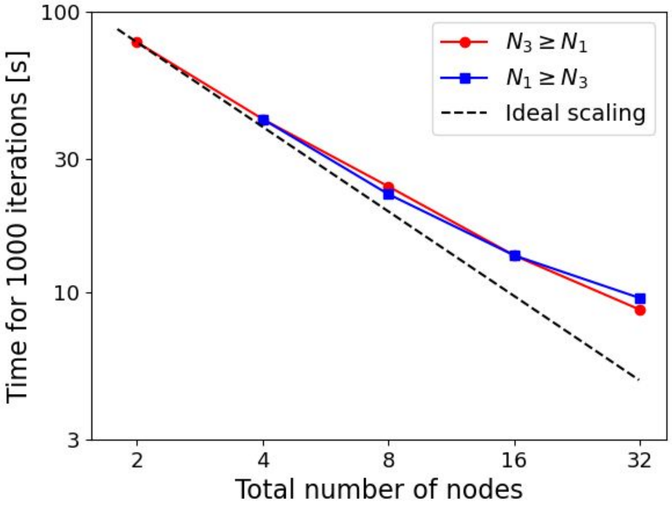
Why a new code?

- All building blocks are well understood:
- Enables modifications
- Enables adding new physics
- High efficiency.

Bégué, Pe'er et al. (2023) ; Zhang, Bégué, Pe'er et al. (2024) ; Singh, Bégué and Pe'er (ApJL 2025) ; Singh, Bégué and Pe'er (2025, in prep)

Scaling





Strong scaling 512 x 256 x 512

How about radiation ?

General relativistic radiation MHD

What happens when radiation affects the dynamics ?

- Quasars, ULX
- Supernovae explosion (neutrino)
- GRB jets (neutrino vs magnetic)

The radiative dynamical equations becomes:

$$(\rho u^{\mu})_{;\mu} = 0,$$

$$(T^{\mu}_{\nu})_{;\mu} = G_{\nu},$$

$$(R^{\mu}_{\nu})_{;\mu} = -G_{\nu},$$

where

$$\widehat{R} = \begin{bmatrix} \widehat{E} & \widehat{F}^i \\ \widehat{F}^j & \widehat{P}^{ij} \end{bmatrix}$$

$$\widehat{E} = \int \widehat{I}_{\nu}\, \mathrm{d}\nu\, \mathrm{d}\Omega,$$

$$\widehat{F}^i = \int \widehat{I}_{\nu}\, \mathrm{d}\nu\, \mathrm{d}\Omega\, N^i,$$

$$\widehat{P}^{ij} = \int \widehat{I}_{\nu}\, \mathrm{d}\nu\, \mathrm{d}\Omega\, N^i N^j$$

Sadowski et al. (2013) ; McKinney et al. (2014), Jiang (2021), Asahina et al. (2020)

General relativistic radiation MHD

$$(\rho u^{\mu})_{;\mu} = 0,$$

$$(T^{\mu}_{\ \nu})_{;\mu} = G_{\nu},$$

$$(R^{\mu}_{\ \nu})_{;\mu} = -G_{\nu},$$

In addition:

$$G^{i} = \int (\chi_{\nu} I_{\nu} - \eta_{\nu})\, d\nu\, d\Omega\, N^{i},$$

In the comoving frame: $$\widehat{G} = \begin{bmatrix} \kappa(\widehat{E} - 4\pi\widehat{B}) \\ \chi \widehat{F}^{i} \end{bmatrix}.$$

- Assuming a closure relation gives Pij (E, Fi). (loss of angular information).
- In that case, the specific intensity is not solved for.
- Relative simplicity / numerically cheap

Drawbacks

- Not suitable for characterizing observables (spectrum, light-curve)
- Not suitable for regions in which angular dependence of the radiation field becomes important.

<u>Why not solve for the specific intensity ?</u>

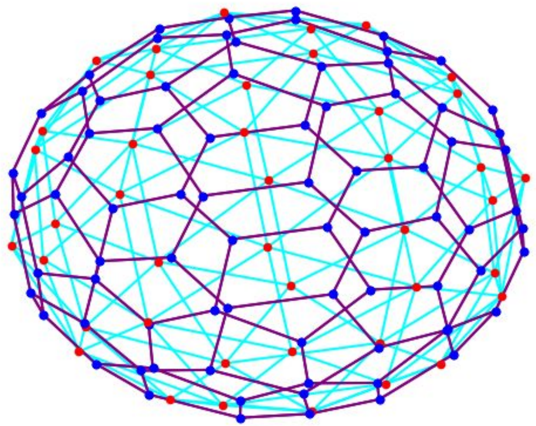
What does it take to solve for Inu ?

We have to resolve everywhere in space, the angular and frequency dependent quantity Inu.

$$N_{tot} = N_{x_1} N_{x_2} N_{x_3} N_{angle} N_{frequency}$$

Tractable with GPUs

Angular grid: We use a geodesic grid



Grid level 2: 162 hexagons and pentagons

Randall et al. (2000, 2002)

Pro: Isotropic discretization of the sphere (avoid pole problem).

Con: Slightly complicated bookkeeping

What does it take to solve for Inu ?

We need an evolutionary equation (radiative transfer equation):

$$\nabla_{\alpha}(\hat{n}^{\alpha}\hat{n}_{\beta}\hat{I}_{\nu}) + \partial_{\hat{\nu}}(\hat{n}^{\hat{\nu}}\hat{n}_{\beta}\hat{I}_{\nu}) + \hat{s}^{-1}\partial_{\hat{\zeta}}(\hat{s}\hat{n}^{\hat{\zeta}}\hat{n}_{\beta}\hat{I}_{\nu}) + \partial_{\hat{\psi}}(\hat{n}^{\hat{\psi}}\hat{n}_{\beta}\hat{I}_{\nu}) = \hat{n}_{\beta}(\hat{j}_{\nu} - \hat{\alpha}_{\nu}\hat{I}_{\nu})$$

Transport term

Gravitational Redshift

Change of direction with the coordinate system

Interaction term

The radiation back reaction is calculated from the variation of the Eddington tensor.

Davis and Gammie (2020), White et al. (2023), Asahina et al. (2020).

What does it take to solve for Inu ?

We need an evolutionary equation (radiative transfer equation):

$$\underbrace{\nabla_\alpha(\hat{n}^\alpha \hat{n}_\beta \hat{I}_\nu) + \partial_{\hat{\nu}}(\hat{n}^{\hat{\nu}} \hat{n}_\beta \hat{I}_\nu) + \hat{s}^{-1}\partial_{\hat{\zeta}}(\hat{s}\hat{n}^{\hat{\zeta}} \hat{n}_\beta \hat{I}_\nu) + \partial_{\hat{\psi}}(\hat{n}^{\hat{\psi}} \hat{n}_\beta \hat{I}_\nu)}_{\text{Transport}} = \underbrace{\hat{n}_\beta(\hat{j}_\nu - \hat{\alpha}_\nu \hat{I}_\nu)}_{\text{Interaction}}$$

In the right (conservative) form to evolve with the finite volume method.

Issue:

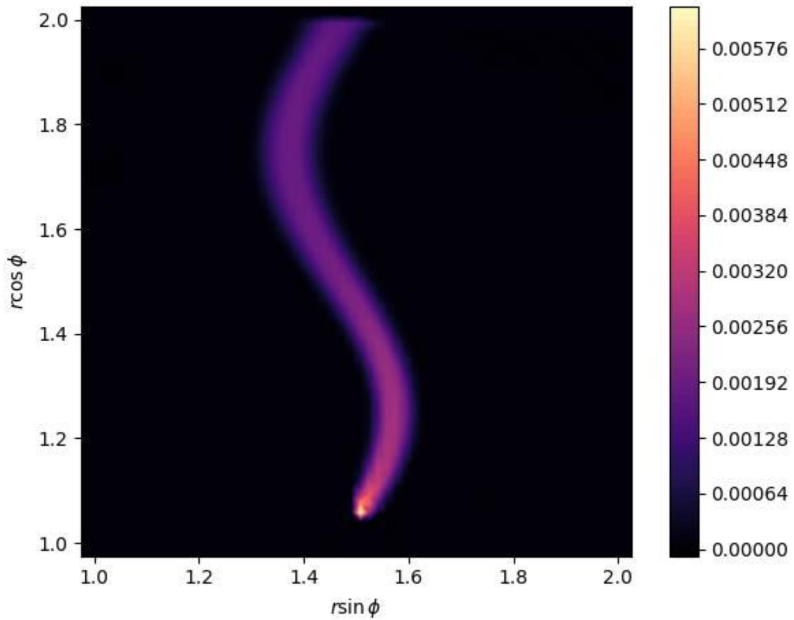
transport and interaction can be on very different time scale.

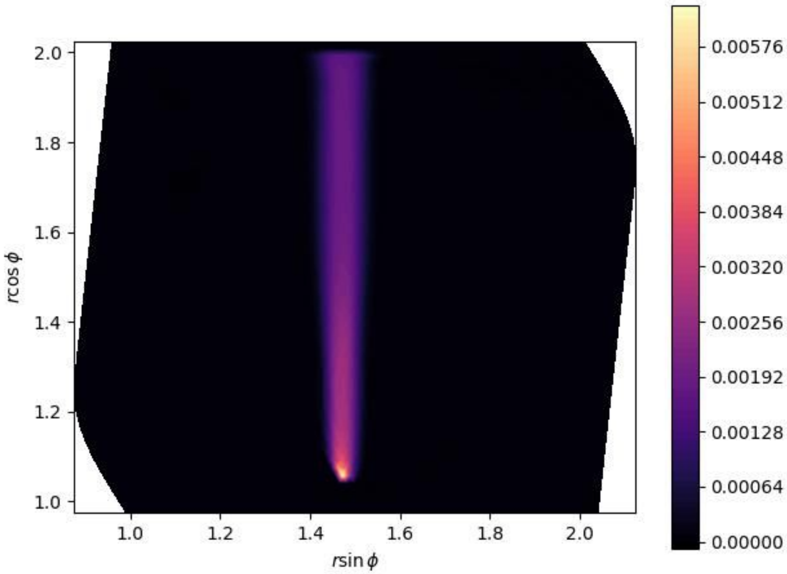
Solution: Operator splitting.

- Explicit transport
- Implicit interaction

Davis and Gammie (2020), White et al. (2023), Asahina et al. (2020).

Just a bit of fun!





Method advantages

Why is it worth paying the price?

1) angular information:
- transition from optically thick to optically thin region
- jetted systems: accurate description of the specific intensity

2) spectral resolution:
- Energy-dependent processes
- Spectral characterisation of the source emission

3) non-thermal description of electrons
- Evolution of their distribution functions (Cost dominated by the radiation).

4) Numerical aspect: the code is specialised to GPUs

Three other groups attempting to evolve the specific intensity:

- ➢ Inazuma (Asahina et al. 2020)
- ➢ Athena++ (White et al. 2023)
- ➢ Arepo (Ma et al 2025)

Test simulation

Spatial resolution: 128 x 64 x 64
Angular resolution: 162 angles (G2)

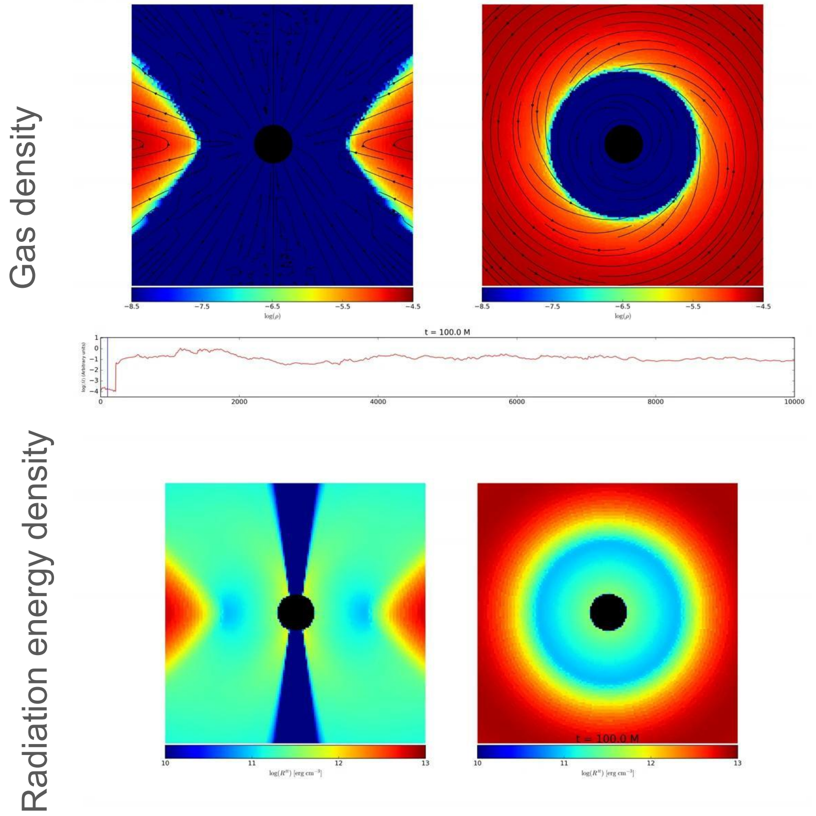
Initial setup:

- Fishbone and Moncrief disk
- R_{in} = 6 ; r_{max} = 12 ;
- Magnetic field: single loop for SANE
- Mass accretion rate: 10^{-2} Medd
- MBH= 10 $M_\odot$
- Spin a = 0.94

Opacities:

- Scattering: κ_s = 0.4 cm^2 g^{-1}
- Absorption: κ_a = 8 x 10^{22} ρ $T^{-3.5}$ cm^2 g^{-1}

We can simulate radiative flows.



Wallace, Bégué, Pe'er (arXiv 2508.15532)

Conclusions

- We wrote a 3D GR-R-MHD code.
- It uses GPUs for accelerating the computation.
- GRMHD module is now being used to study the dynamics of MAD.
- We are finalizing the addition of the radiation sector (Wallace et al., 2025 submitted):
  - Transport ✔
  - Simplified interaction ✔

Next challenges:

- Anisotropic and energy dependent interactions on the geodesic grid (CS, synchroton, ...)
- Electron temperature model: 2-T plasma evolution, proton/electron coupling.
- Numerical optimization of the radation modules.