PHARE: Towards mesh and model refinement for space plasmas

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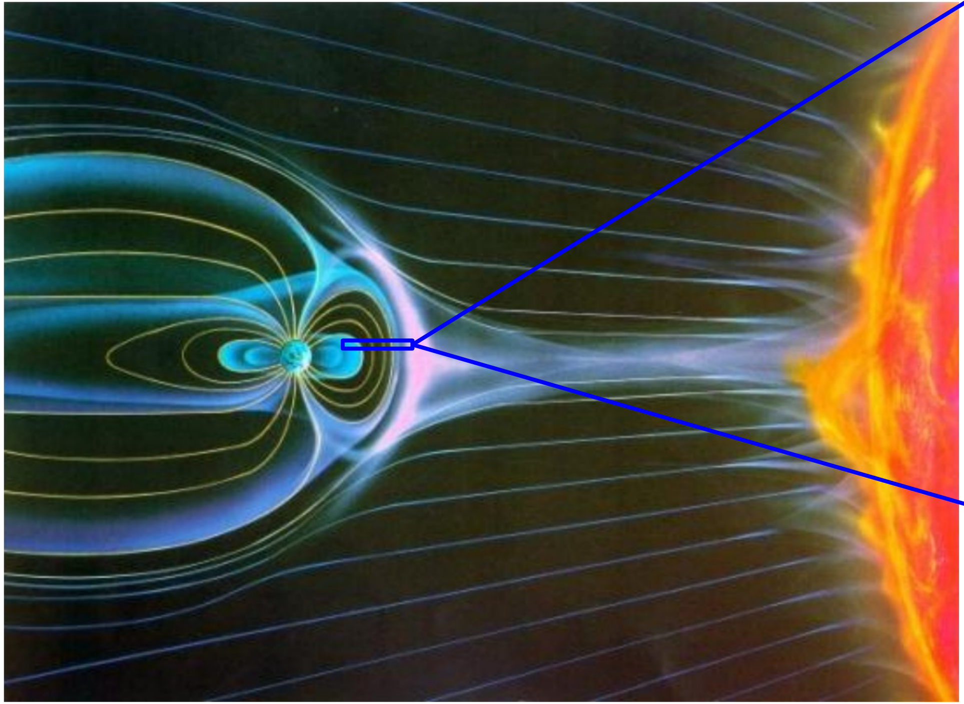
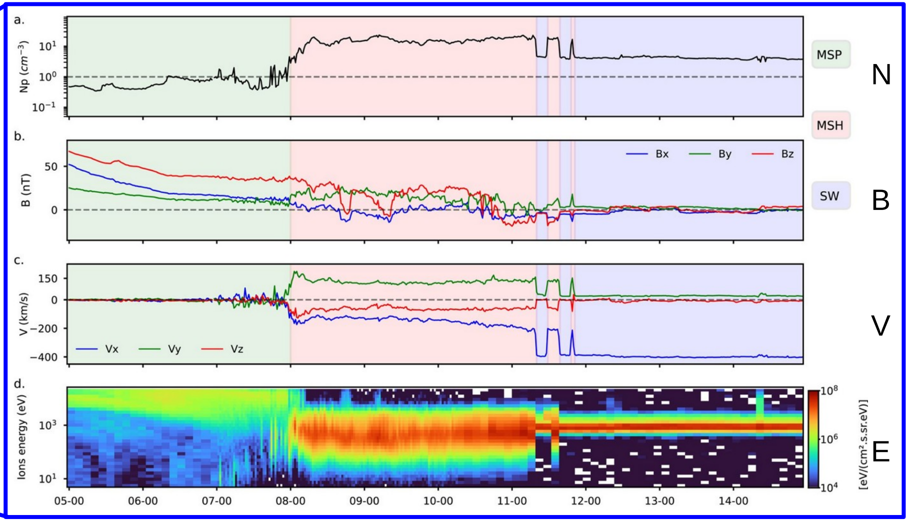
High resolution but localised

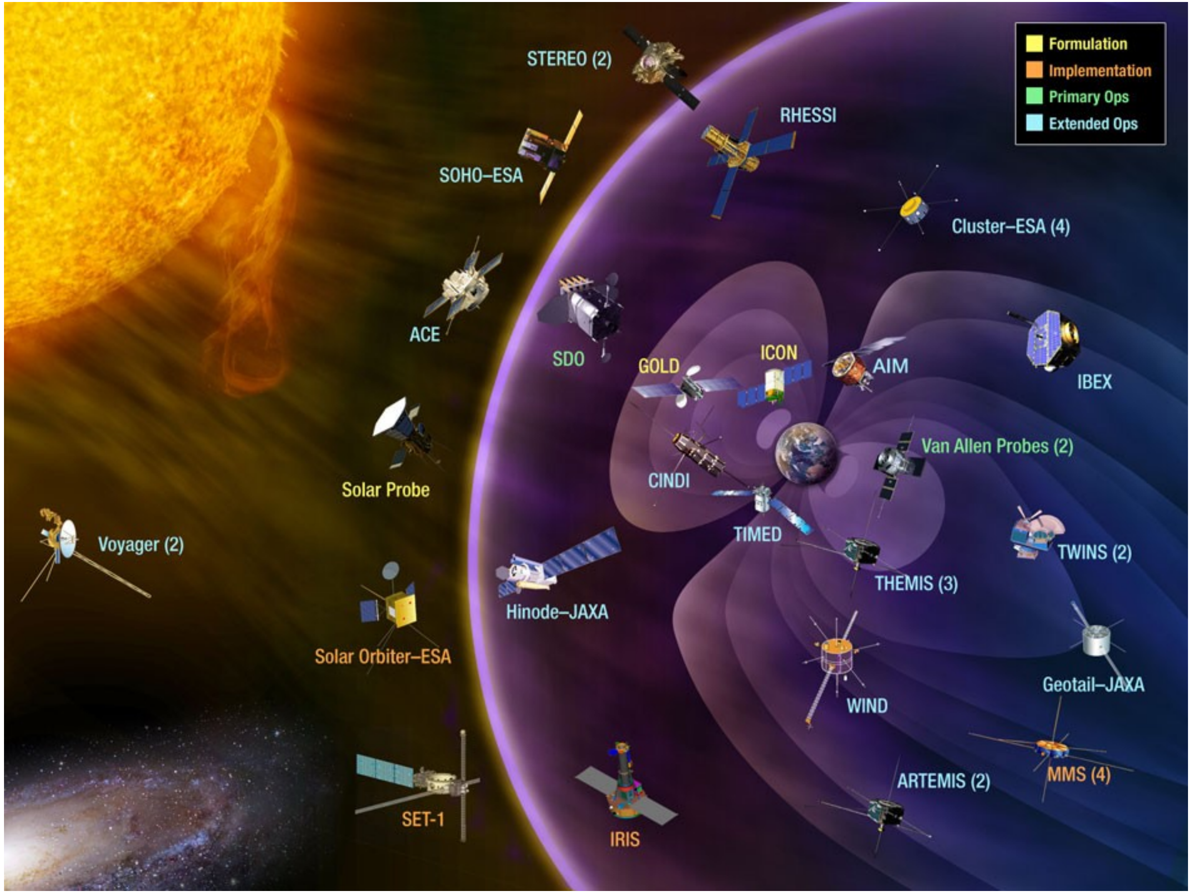




[Michotte de Welle et al. 2022]

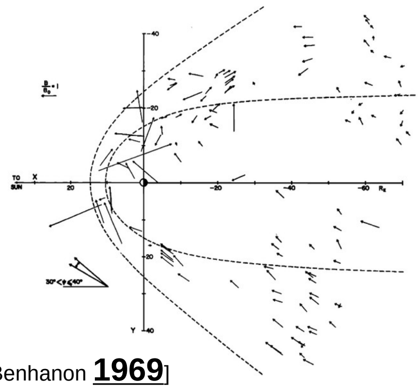
High space/time resolution in but no global vision.

We have decades of good quality data

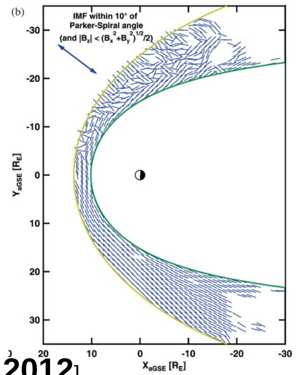


Statistics and machine learning: Global but not dynamic

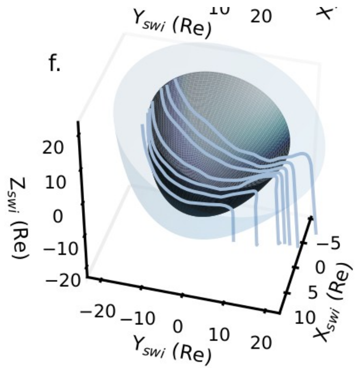
Recent efforts in machine learning has allowed scaling to much larger number of point



[Benhanon **1969**]



[Petrinec **2012**]

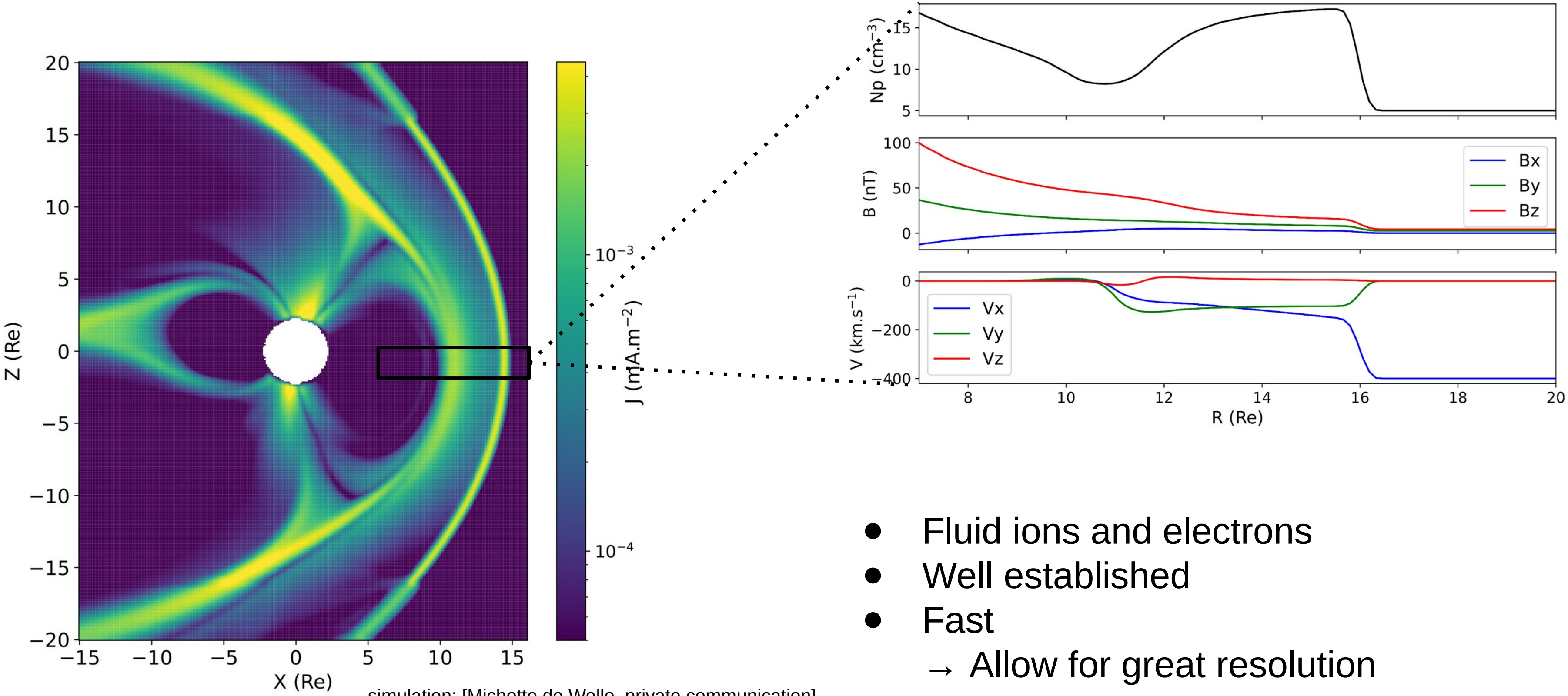


[**Michotte de Welle 2022**]

Averaged: we lose all small (temporal and spatial) scale processes that drive the global dynamics.

We want simulations to have both the global scale and the dynamics.

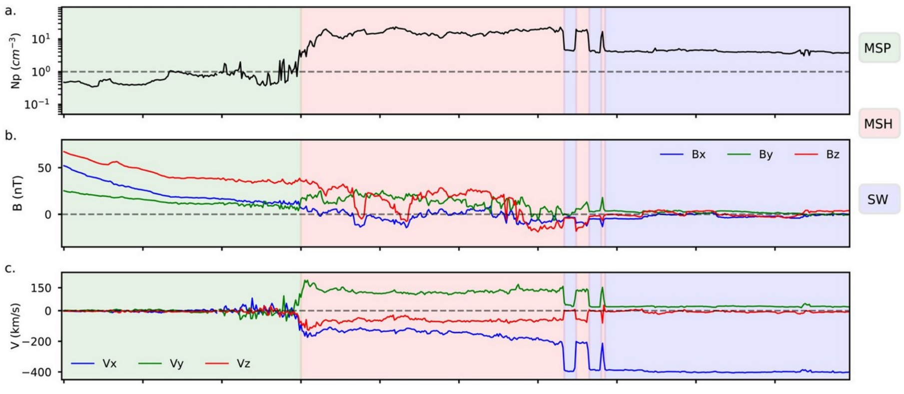
State of the art of global simulations: MHD

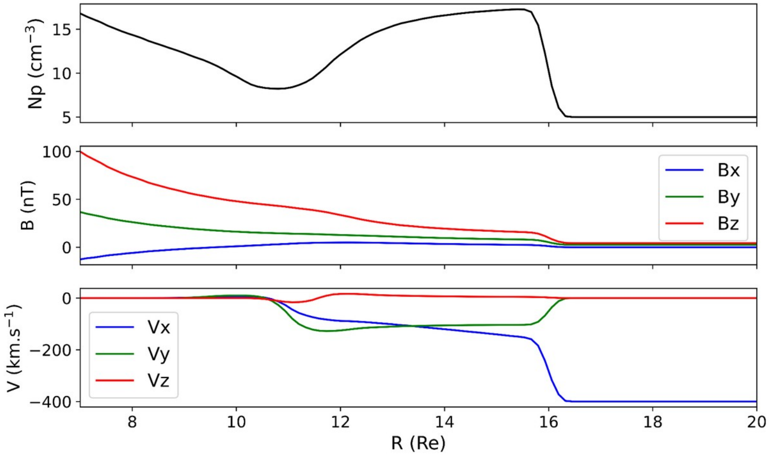


simulation: [Michotte de Welle, private communication]
code BATS-R-US: [Gombosi et al. 2001]

- Fluid ions and electrons
- Well established
- Fast
  → Allow for great resolution

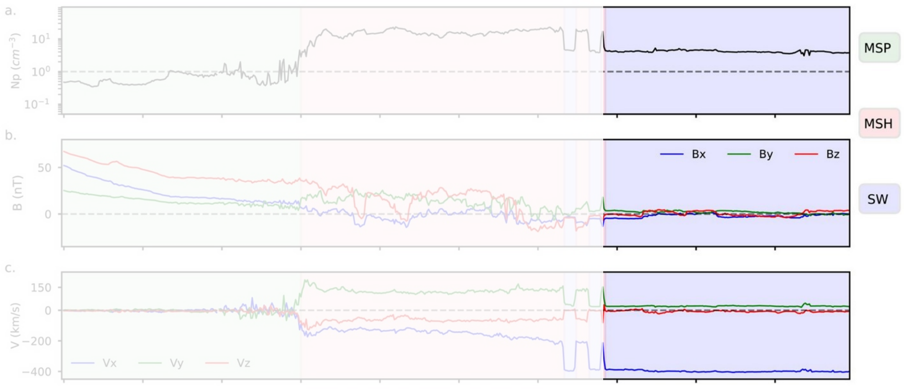
MHD capture regions and their interfaces



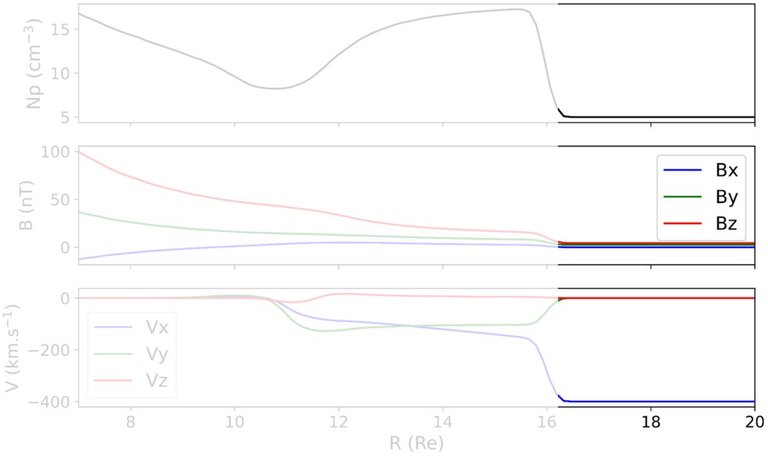


MHD capture regions and their interfaces

Solar wind

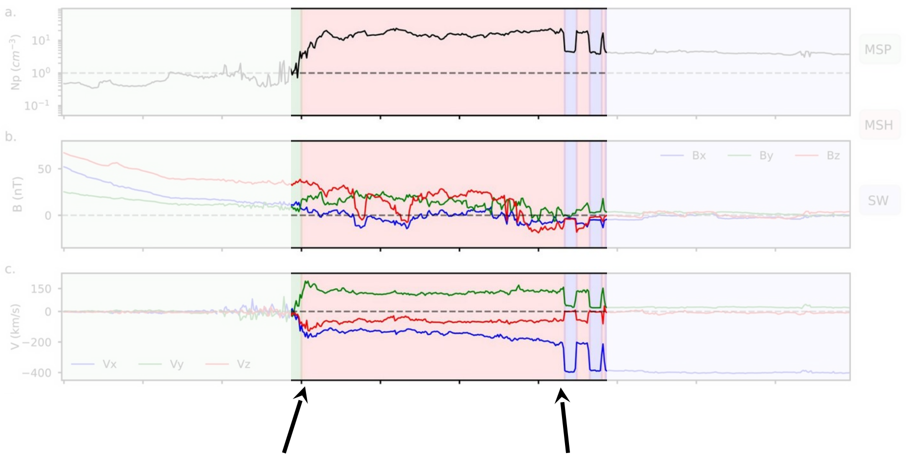


Solar wind



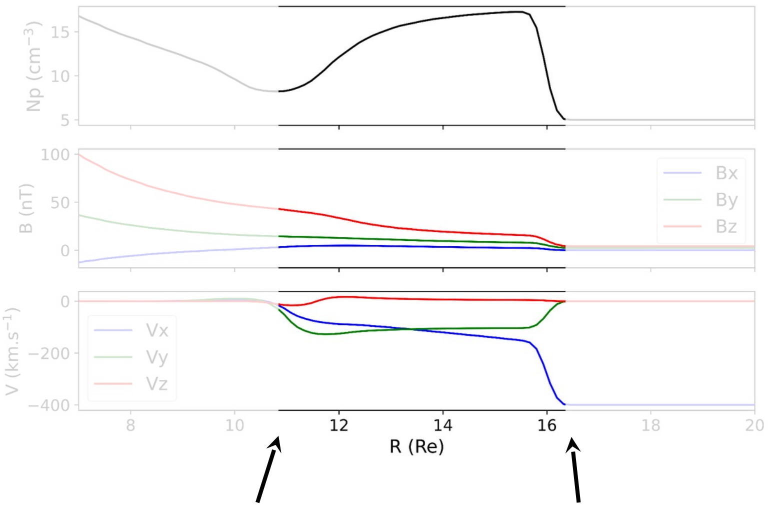
MHD capture regions and their interfaces

Magnetosheath



Magnetopause
Bow shock

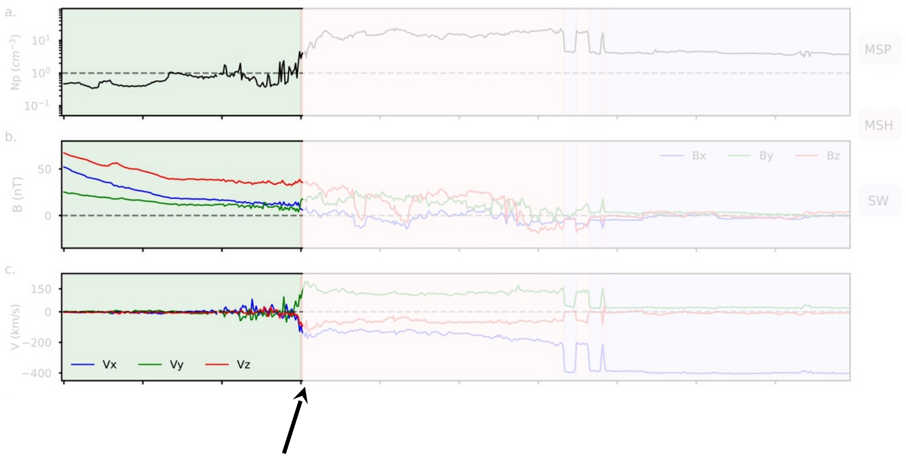
Magnetosheath



Magnetopause
Bow shock

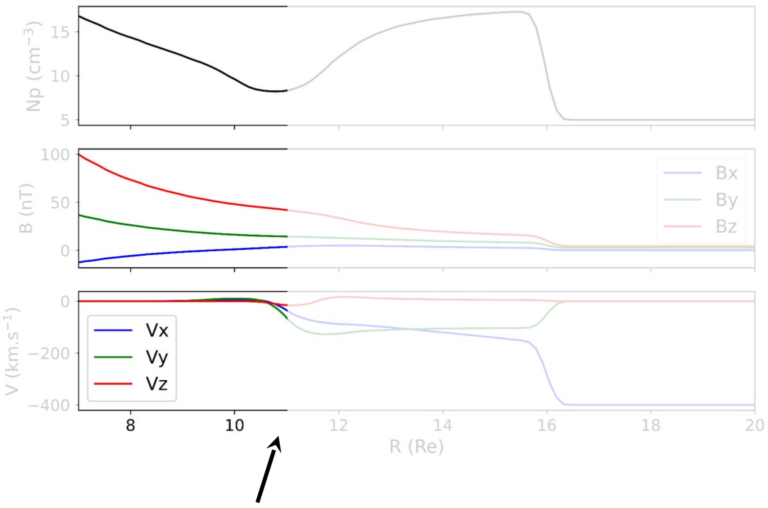
MHD capture regions and their interfaces

Magnetosphere



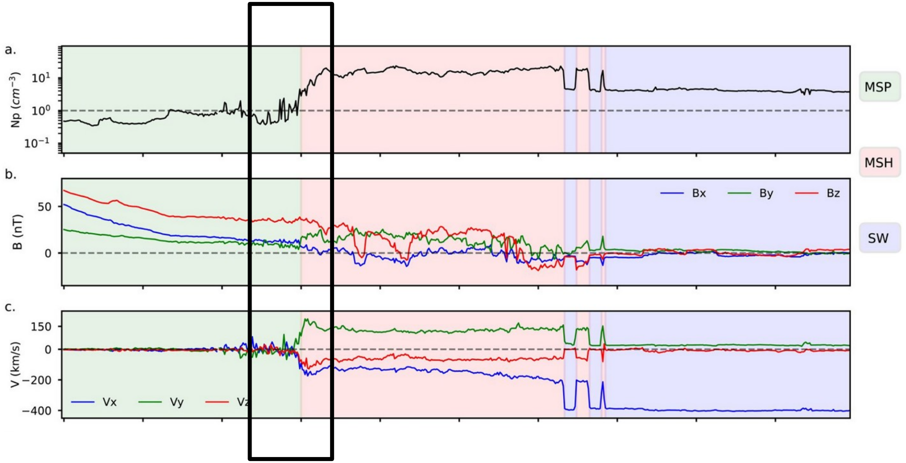
Magnetopause

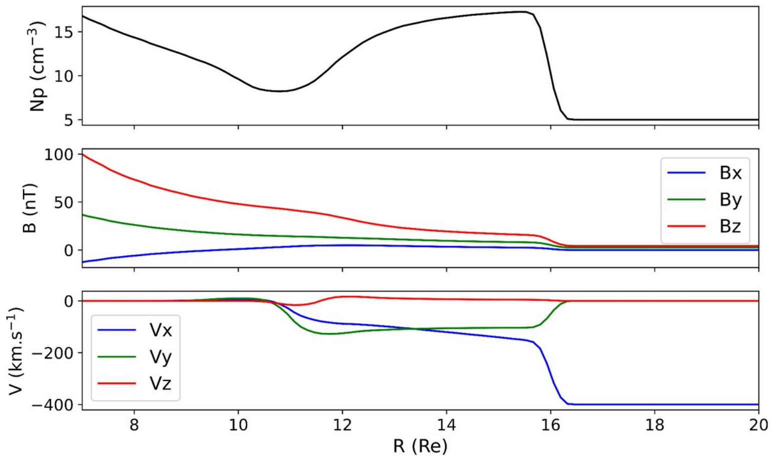
Magnetosphere



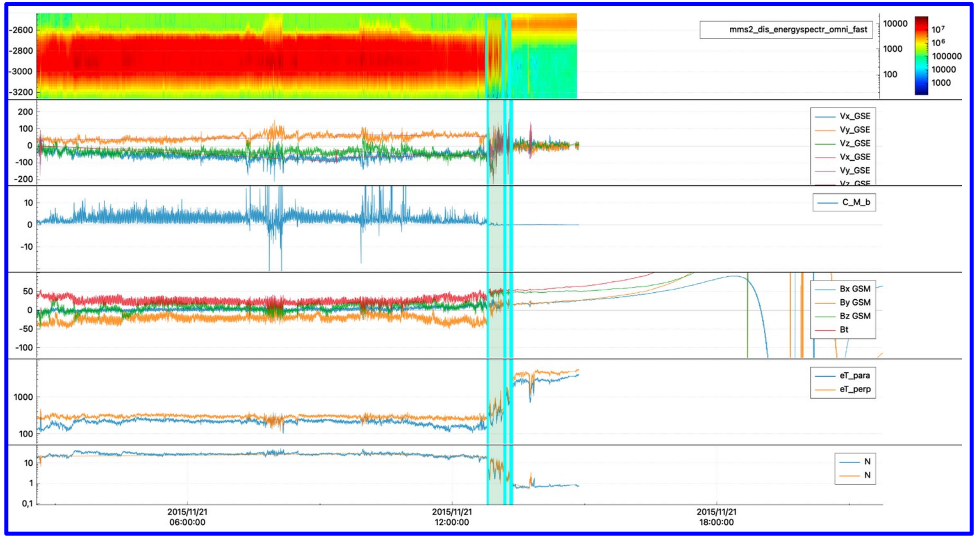
Magnetopause

But misses small-scale processes driving their coupling





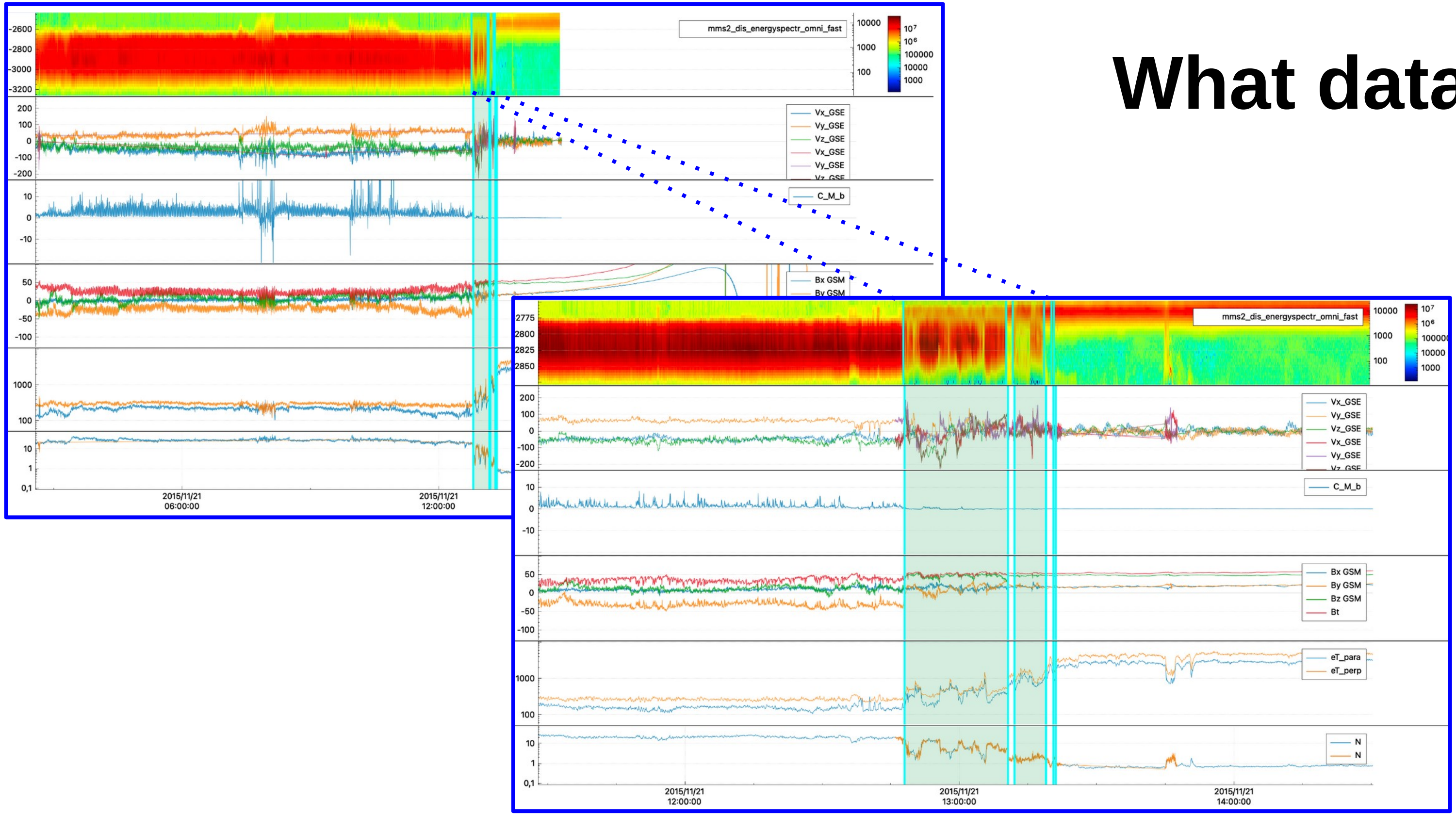
What happens in the active boundaries between the different environments ?



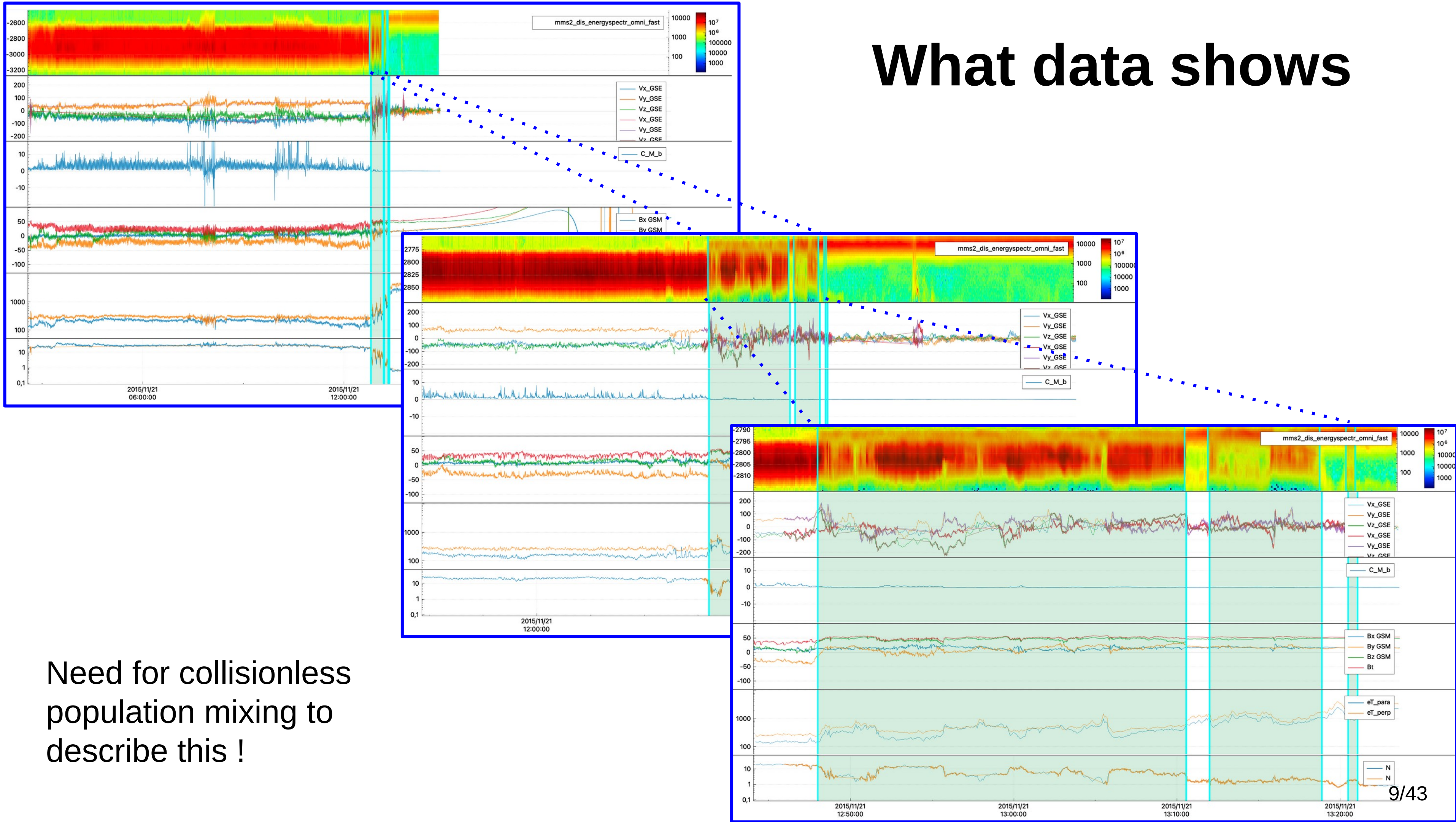
[SciQLop software: Jeandet et al. (2025)]
https://doi.org/10.5281/zenodo.17176824

What data shows

What data shows

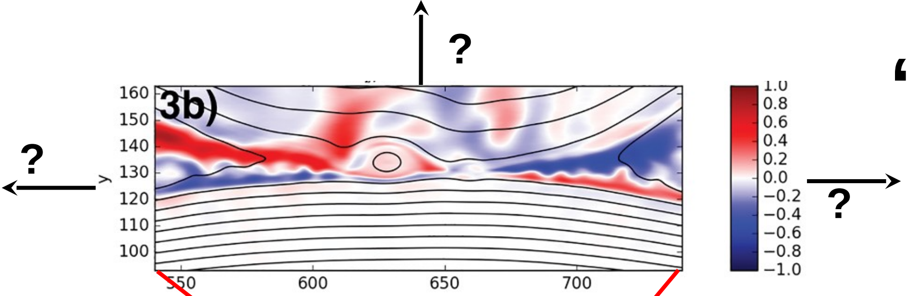


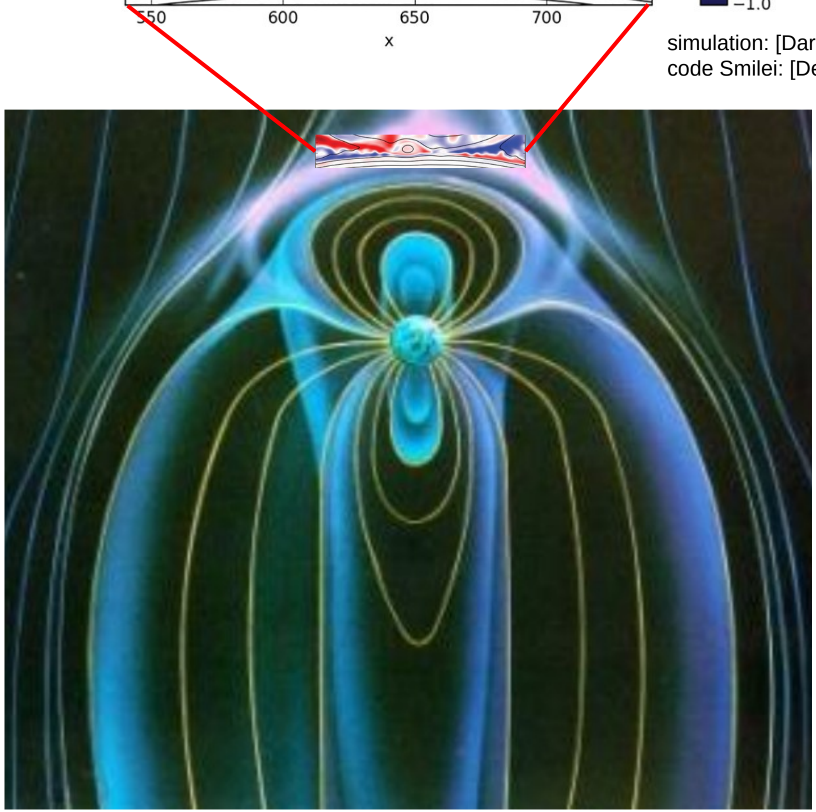
What data shows



Need for collisionless population mixing to describe this !

“Full” PIC: no global scale

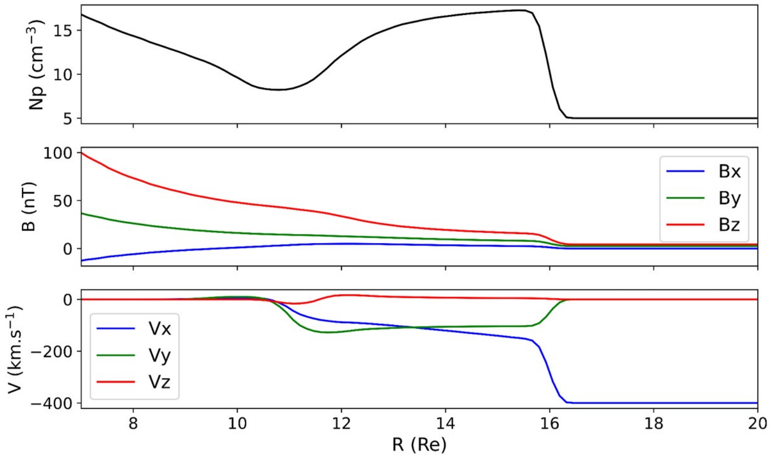




simulation: [Dargent et al. 2020]
code Smilei: [Derouillat et al. 2018]

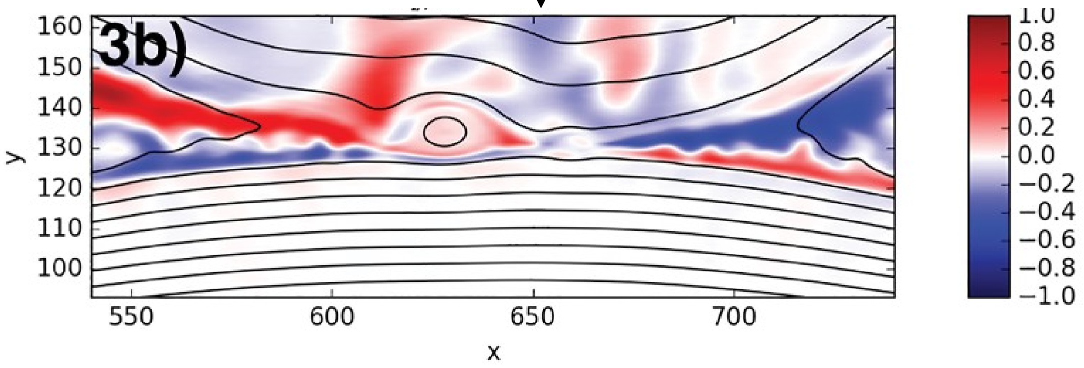
- Kinetic ions and electrons
- Realistic physics
- Very computationally heavy
  →Restrained to a small box
  →Global scale is not attainable

How can we do better ?

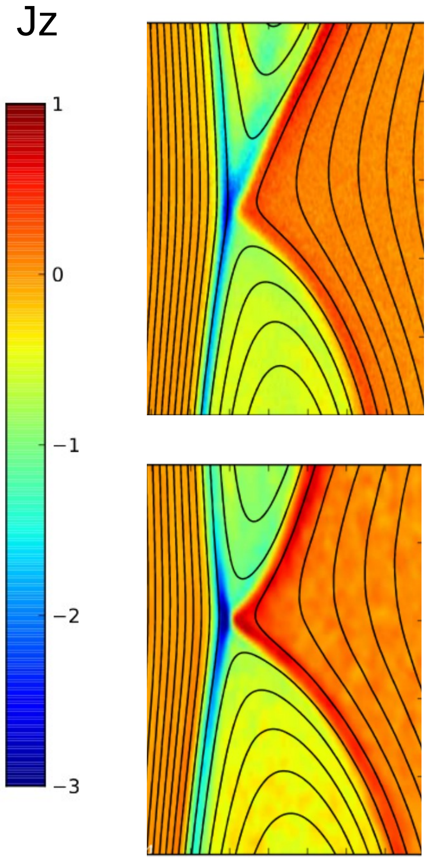


No microphysics !

No global scales !



Hybrid PIC: the good compromise ?



Full PIC

Hybrid PIC

[Aunai et al. 2013]

- Kinetic ions, **<u>fluid electrons</u>**
- Physically realistic if well resolved

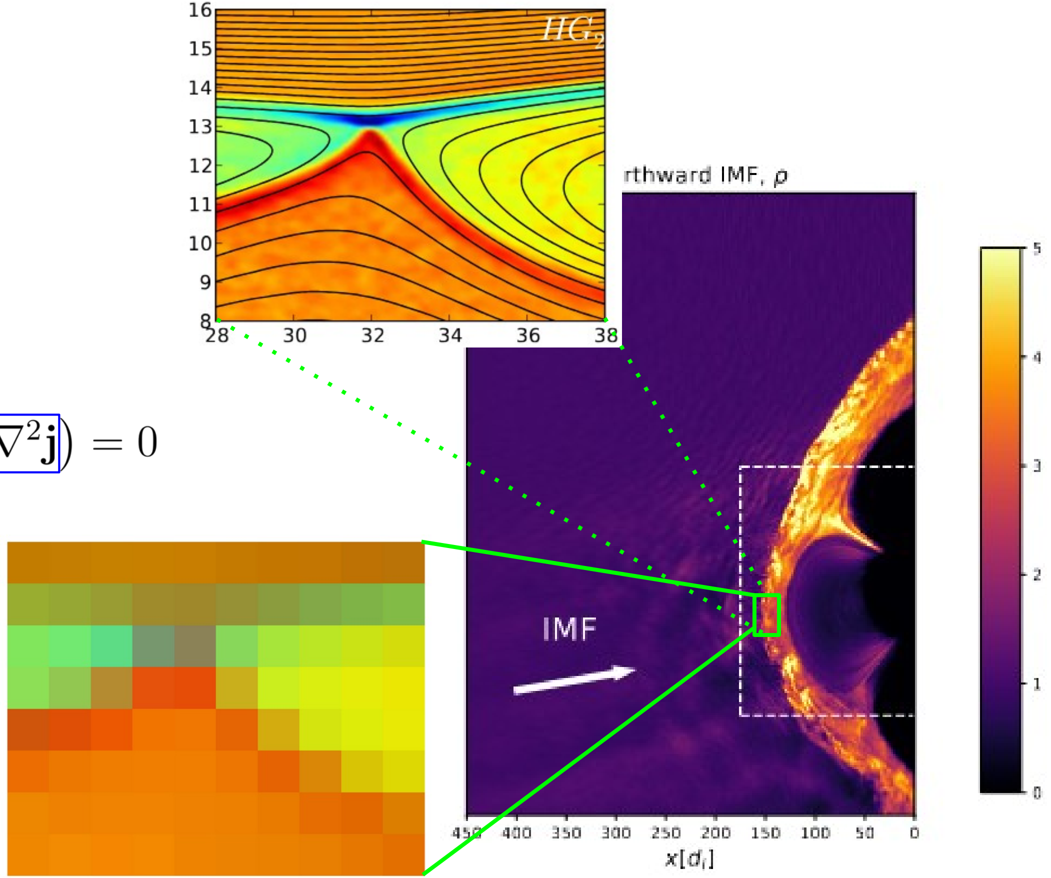
→Global simulations attainable ?

Hard to have both small and large scales

no large scale

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \left(-\mathbf{v}_i \times \mathbf{B} + \eta \mathbf{j} + \frac{\mathbf{j}\times\mathbf{B} - \nabla P_e}{en} - \nu \nabla^2 \mathbf{j} \right) = 0$$

unresolved small scales



simulation: [Ng et al. 2022]
code HYPERS: [Omelchenko & Karimabadi 2012]

We can’t just “use bigger computers”

Resolution in 3d hybrid simulations:

Goal:

$$1 \text{ to } 5\delta_i \longrightarrow 0.05\delta_i \qquad 100$$

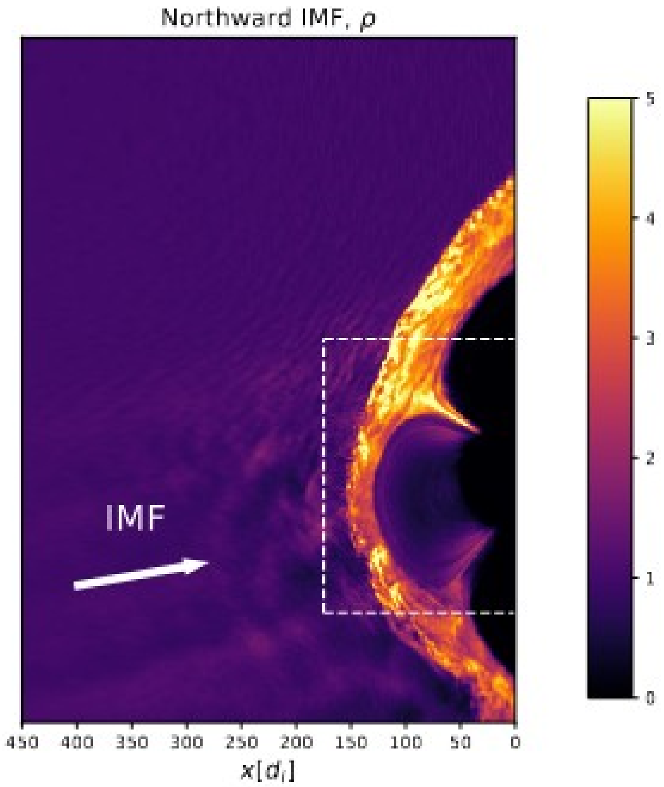
Per dimension: $100^3 = 10^6$

$\times$

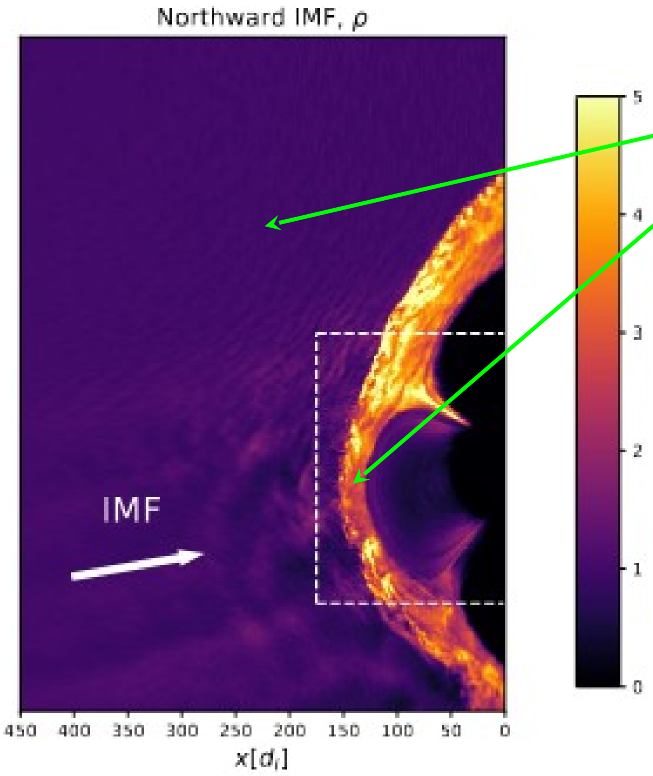
Time step: $\Delta t \sim \Delta x^2$ $\qquad 100^2 = 10^4$

$$100^3 \times 100^2 = 10^{10}$$

We would need 10^{10} times more compute power !



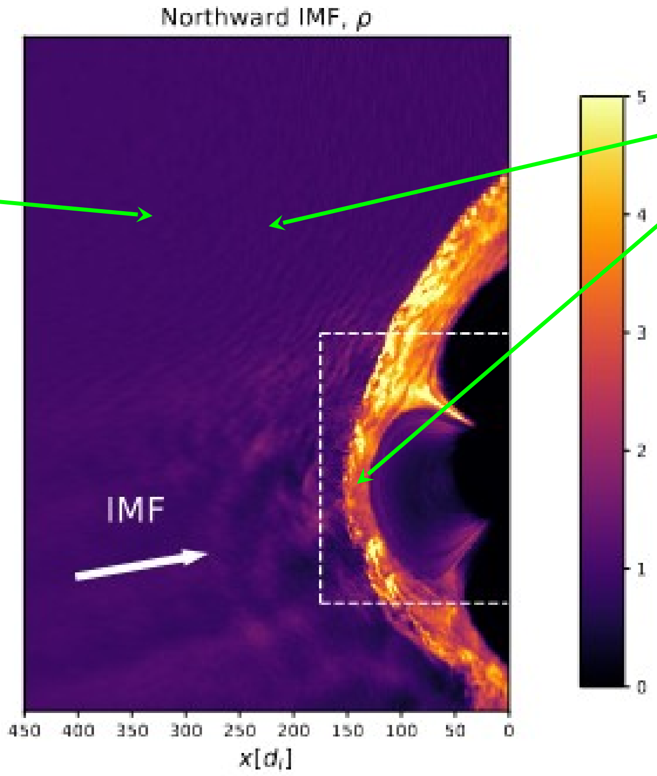
We need a different approach



Same mesh resolution ?

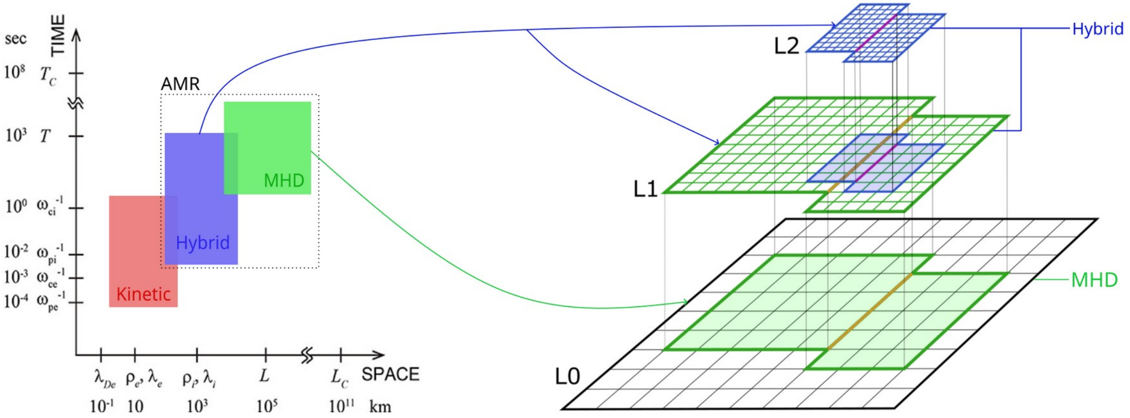
We need a different approach

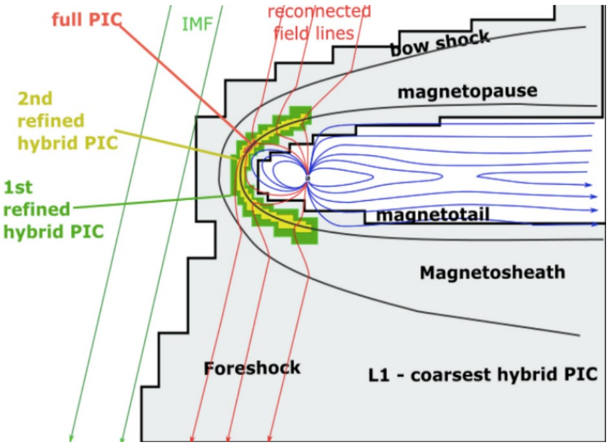
Do we need hybrid there ?



Same mesh resolution ?

Our goal in PHARE: adaptive mesh and model refinement (AM2R)





PHARE: an open-source, modern C++ plasma simulation code developed at LPP **for the community**.

A code for the community

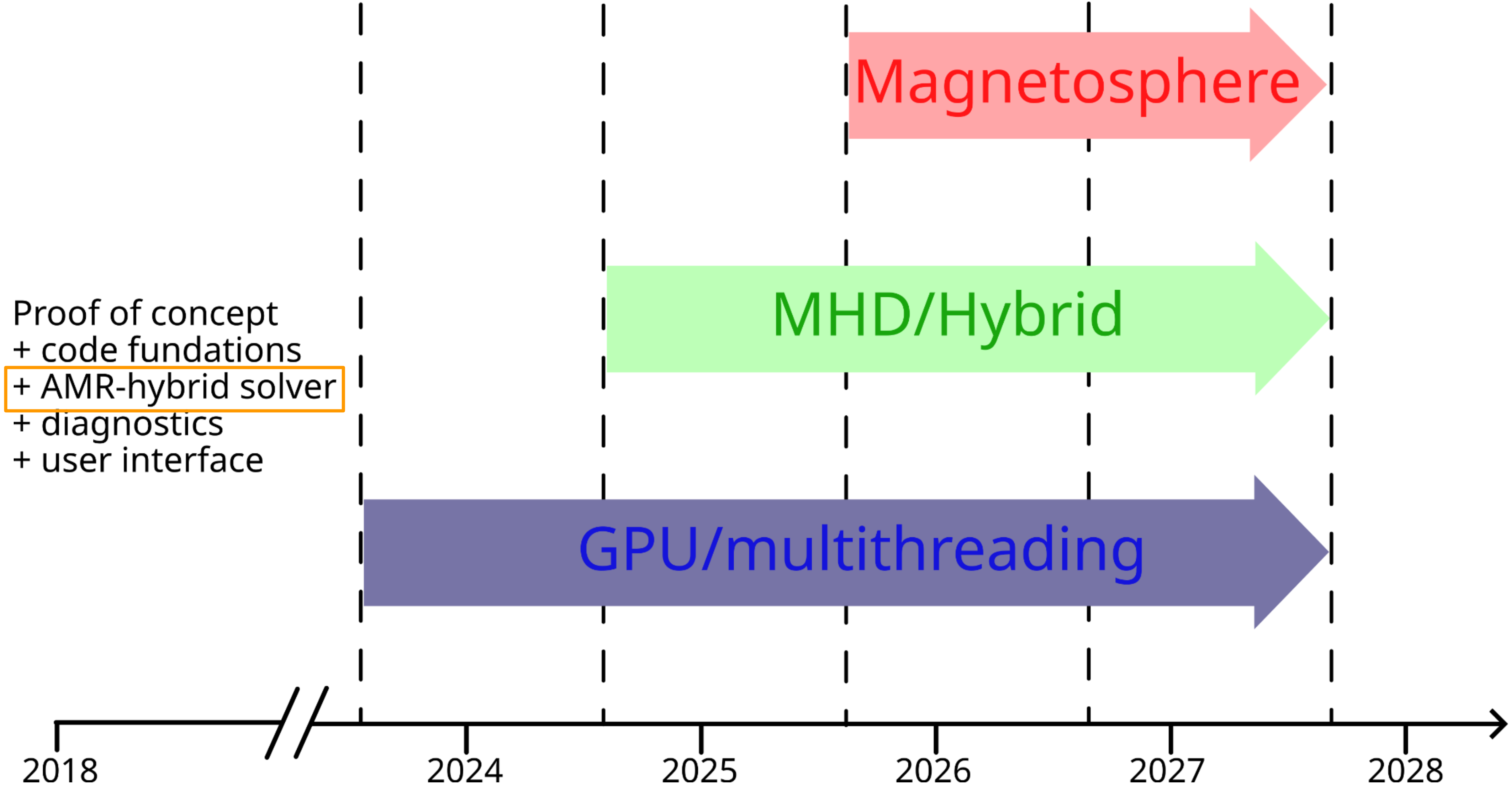
Open source

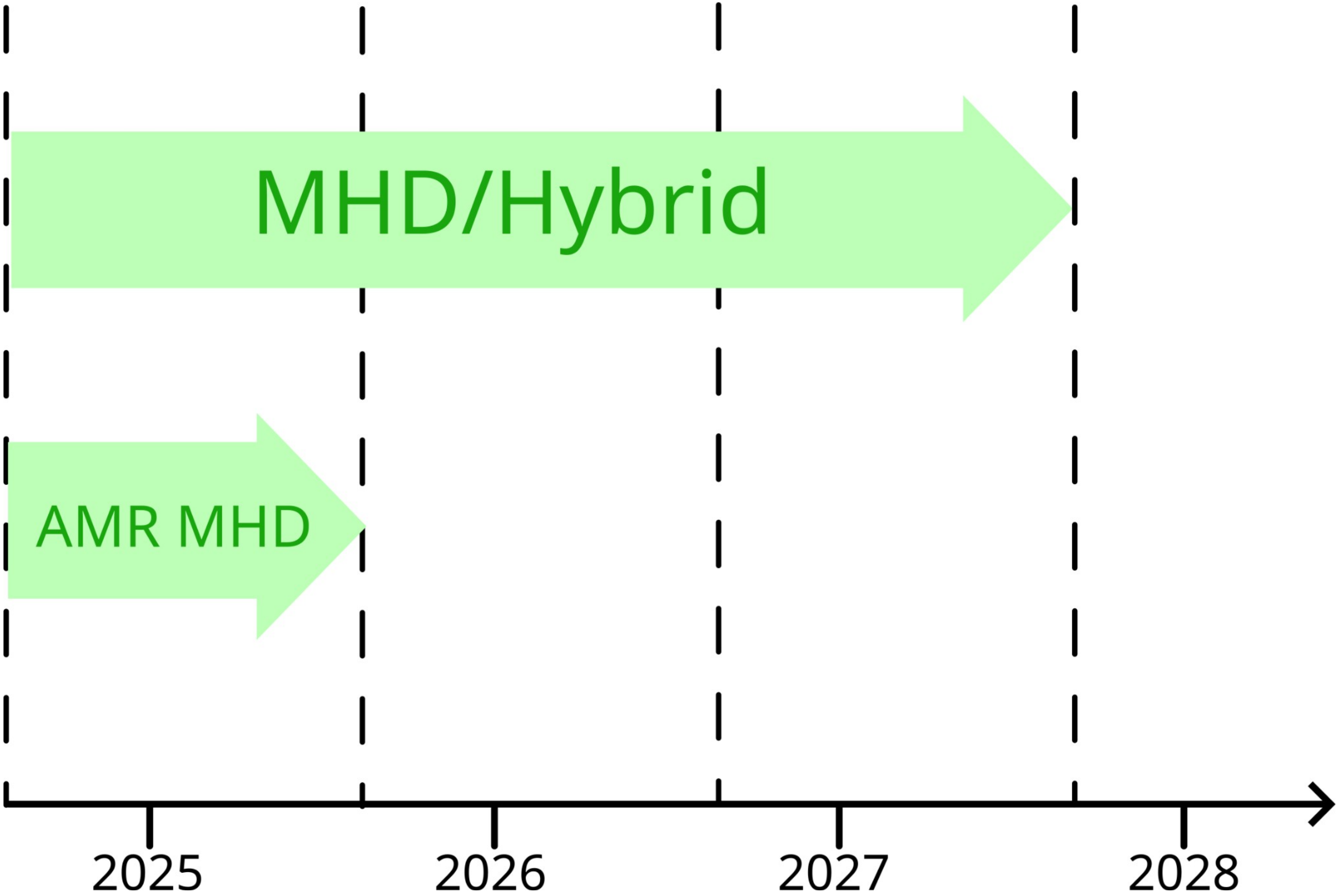
High perf,
abstractions

User friendly

Testability,
Robustness

- 100% open source.
- Modern c++ (c++20) for performance and abstraction while maintaining friendly python user interface.
- Extensive testing running on continuous integrations.
- Relies on well established AMR library called SAMRAI.


Magnetosphere
MHD/Hybrid
GPU/multithreading
Proof of concept
+ code fundations
+ AMR-hybrid solver
+ diagnostics
+ user interface
2018
2024
2025
2026
2027
2028


MHD/Hybrid
AMR MHD
2025
2026
2027
2028

Step 1: choosing the equations

We advance the following equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Continuity equation

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot [\rho \mathbf{v}\mathbf{v} - \mathbf{B}\mathbf{B} + P^{*}] = 0$$

Momentum equation

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla \cdot \left[(\mathcal{E} + P^{*})\mathbf{v} + \frac{\mathbf{E}\times\mathbf{B}}{\mu_0} \right] = 0$$

Energy equation

with:

$$P^{*} = P + \frac{\mathbf{B}\cdot\mathbf{B}}{2}$$

The total pressure

Step 1: choosing the equations

We advance the following equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Continuity equation

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot [\rho \mathbf{v}\mathbf{v} - \mathbf{B}\mathbf{B} + P^*] = 0$$

Momentum equation

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla \cdot \left[(\mathcal{E} + P^*)\mathbf{v} + \frac{\mathbf{E}\times\mathbf{B}}{\mu_0} \right] = 0$$

Energy equation

Polytropic closure:

$$\mathcal{E} = \frac{P}{\gamma - 1} + \frac{1}{2}\left(\rho(\mathbf{v} \cdot \mathbf{v}) + \frac{\mathbf{B}\cdot\mathbf{B}}{\mu_0}\right)$$

Step 1: choosing the equations

We advance the following equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Continuity equation

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot [\rho \mathbf{v}\mathbf{v} - \mathbf{B}\mathbf{B} + P^*] = 0$$

Momentum equation

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla \cdot \left[(\mathcal{E} + P^*)\mathbf{v} + \frac{\mathbf{E}\times\mathbf{B}}{\mu_0} \right] = 0$$

Energy equation

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0$$

Faraday's law

Hall | Ohmic resistivity | Hyper-resistivity

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \frac{\mathbf{j}\times\mathbf{B}}{e n_e} + \eta \mathbf{j} - \nu \nabla^2 \mathbf{j}$$

Ohm's law

with:

$$\mathbf{j} = \frac{\nabla\times\mathbf{B}}{\mu_0}$$

Ampere's law

Step 1: choosing the equations

We advance the following equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$ Continuity equation

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot [\rho \mathbf{v}\mathbf{v} - \mathbf{B}\mathbf{B} + P^*] = 0$$ Momentum equation

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla \cdot \left[(\mathcal{E} + P^*)\mathbf{v} + \frac{\mathbf{E} \times \mathbf{B}}{\mu_0} \right] = 0$$ Energy equation

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0$$ Faraday’s law

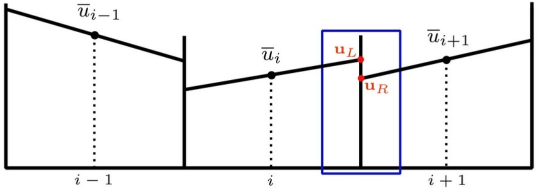
Under the constraint:

$$\nabla \cdot \mathbf{B} = 0$$

Step 2: Numerical implementation

Numerical scheme: Godunov Finite Volumes

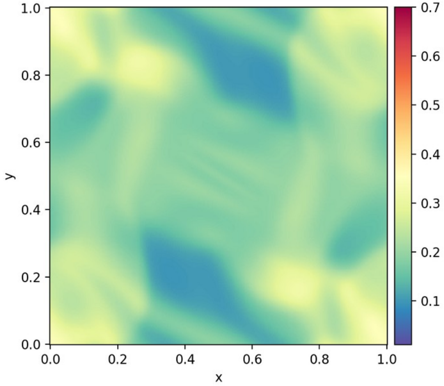
[Godunov 1959]



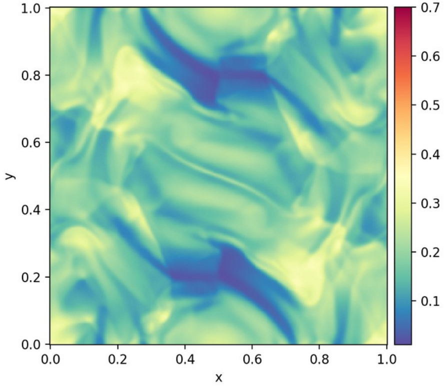
Reconstruction step

Pressure on Orszag-Tang vortex 256x256 at t=1.

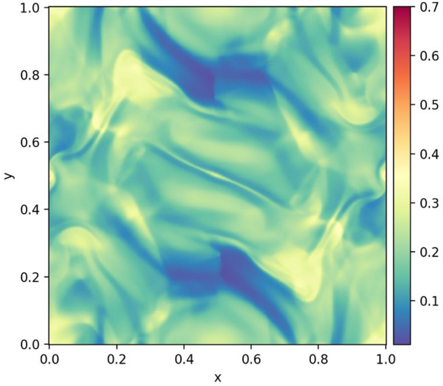
Constant



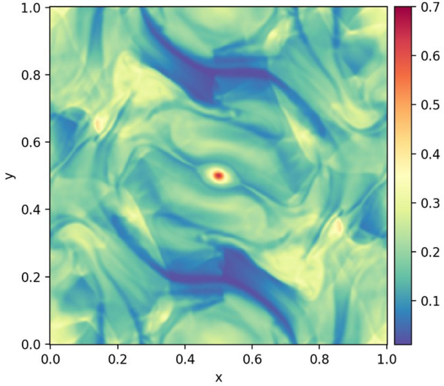
Linear



WENO3



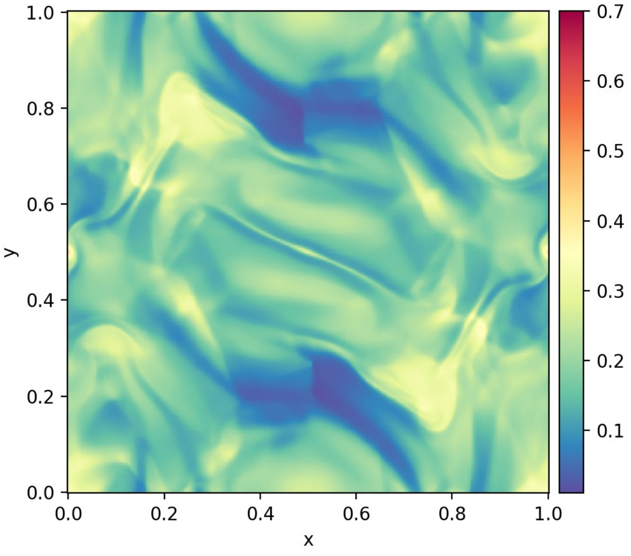
WENOZ



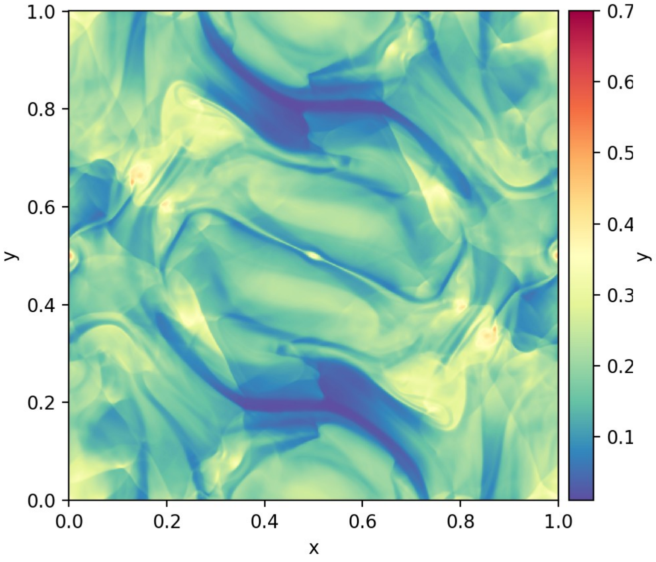
[Orszag & Tang 1979]

Functional Test: Orszag-Tang vortex

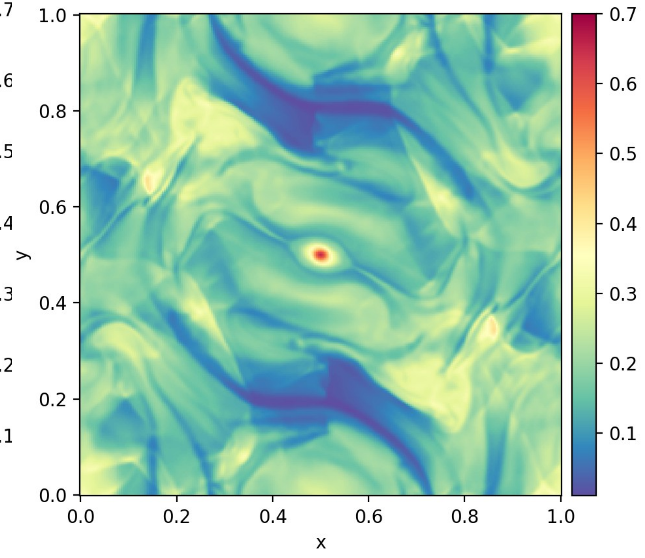
WENO3 256x256



WENO3 1024x1024



WENOZ 256x256



Riemann Solvers

Ideal MHD

Hall MHD

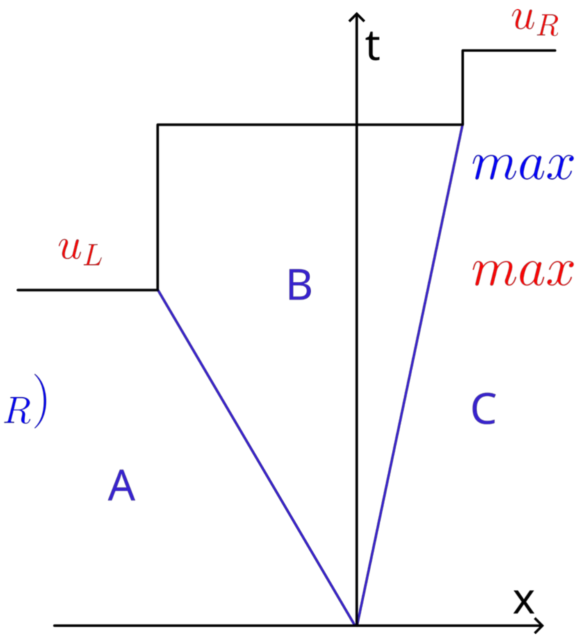
Rusanov,
HLL

$$max(v_L + c_{fast,L}, v_R + c_{fast,R})$$

$$max(v_L + c_{w,L}, v_R + c_{w,R})$$

$$min(v_L - c_{fast,L}, v_R - c_{fast,R})$$

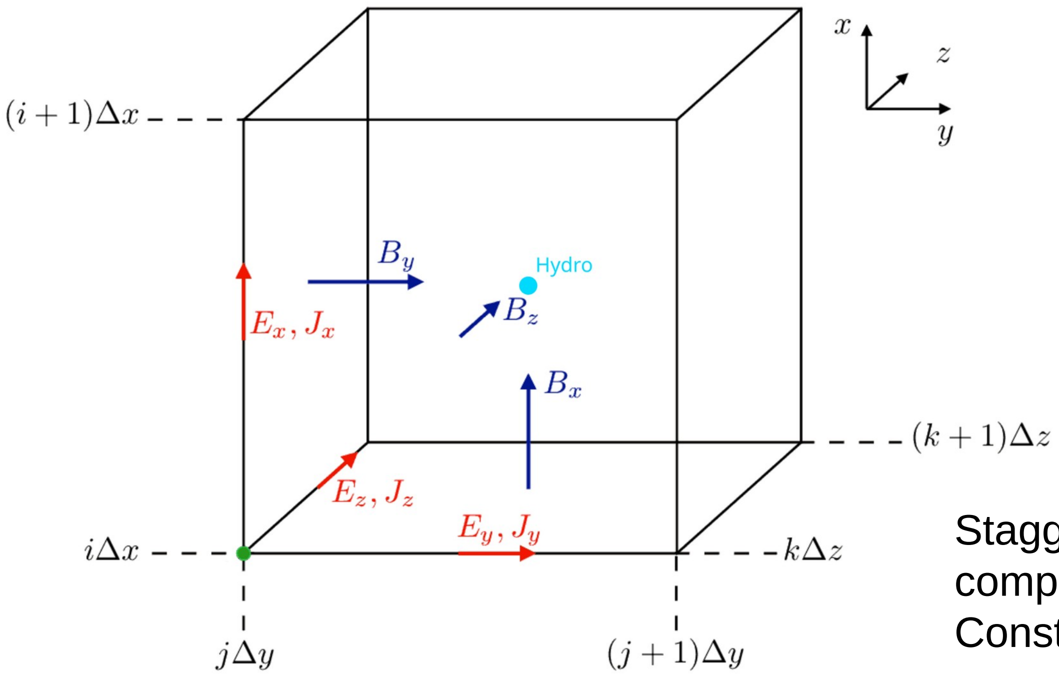
$$min(v_L - c_{w,L}, v_R - c_{w,R})$$



c_{fast} = fast magnetosonic speed

c_w = whistler speed

Maintaining divergence free condition



[Yee 1966]

$$\nabla \cdot \mathbf{B} = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0$$

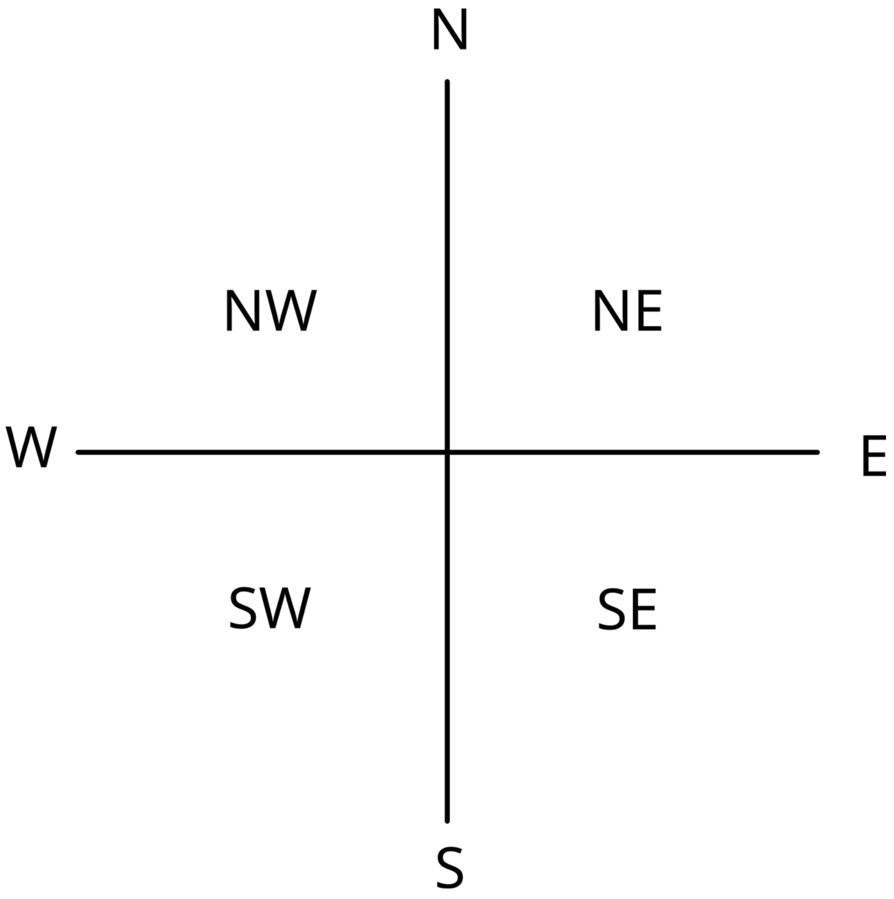
Staggered grid for **B, E, J**. Special care needed to compute **E**, using a class of schemes called Constrained Transport (CT).

In PHARE: CT averaged, Upwind CT.

[Mignone & Del Zanna 2021]

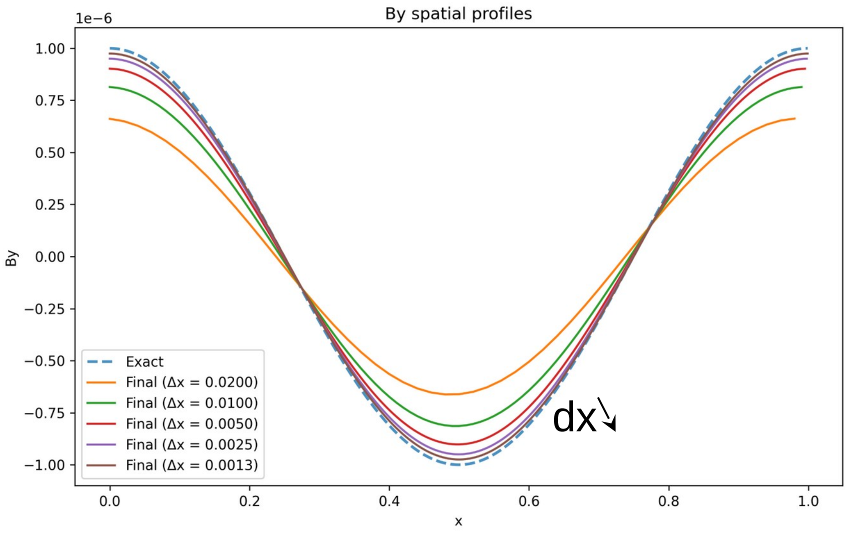
Upwind Constrained Transport (UCT)

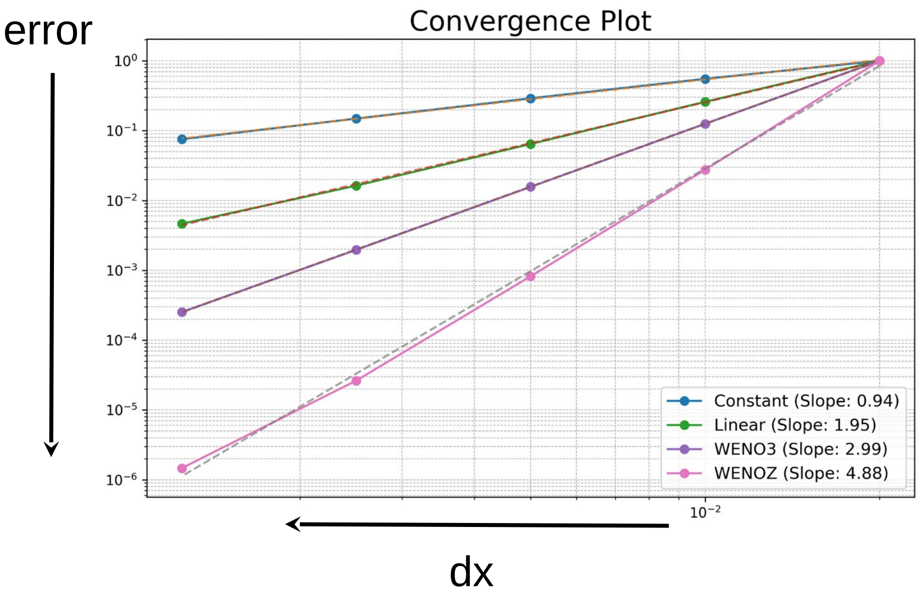
[Mignone & Del Zanna 2021]



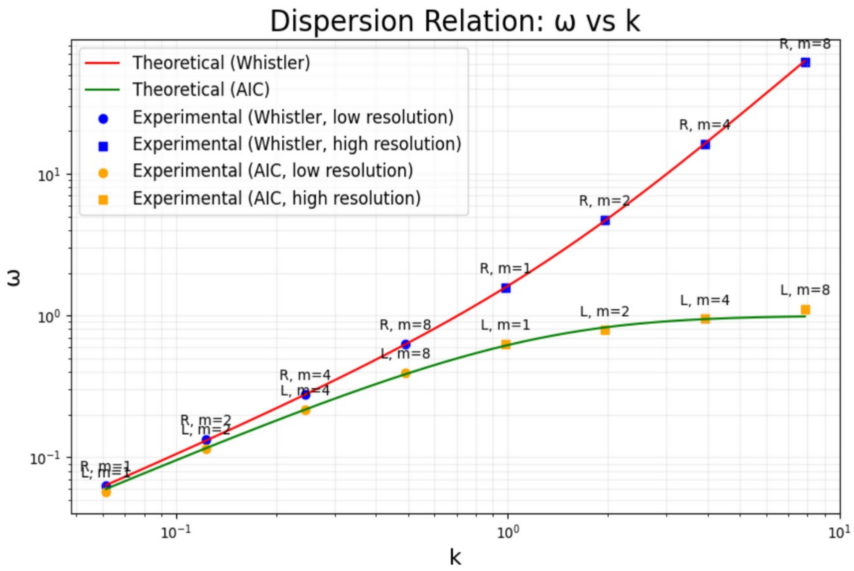
- Reconstruction on the edge directly
- 2d Riemann problem solve as opposed to arithmetic averaging of 1d riemann solvers

Theoretical Validation: Convergence





Theoretical Validation: Hall MHD dispersion



Alfvén Ion Cyclotron:

$$\omega_L = \frac{k^2}{2}\left(\sqrt{1+\frac{4}{k^2}} - 1\right)$$

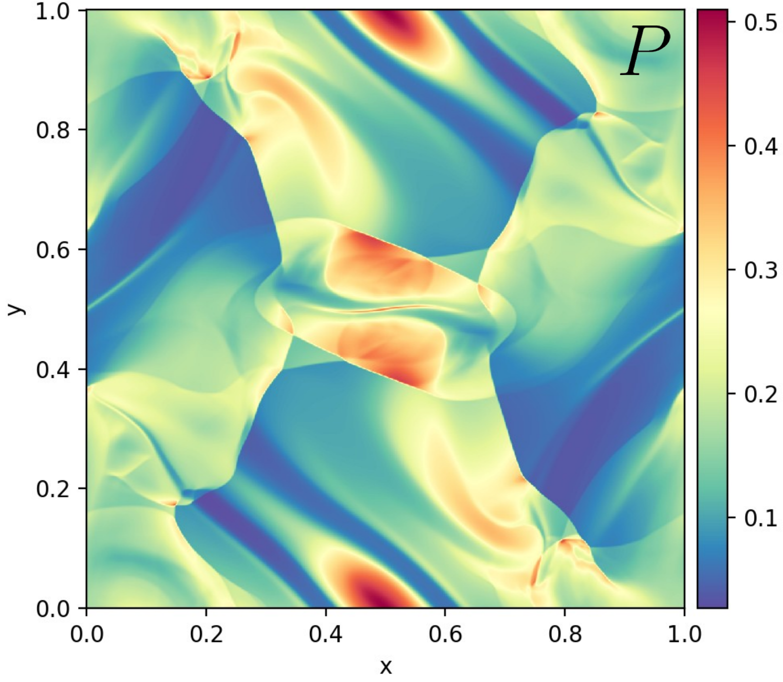
Whistler:

$$\omega_R = \frac{k^2}{2}\left(\sqrt{1+\frac{4}{k^2}} + 1\right)$$

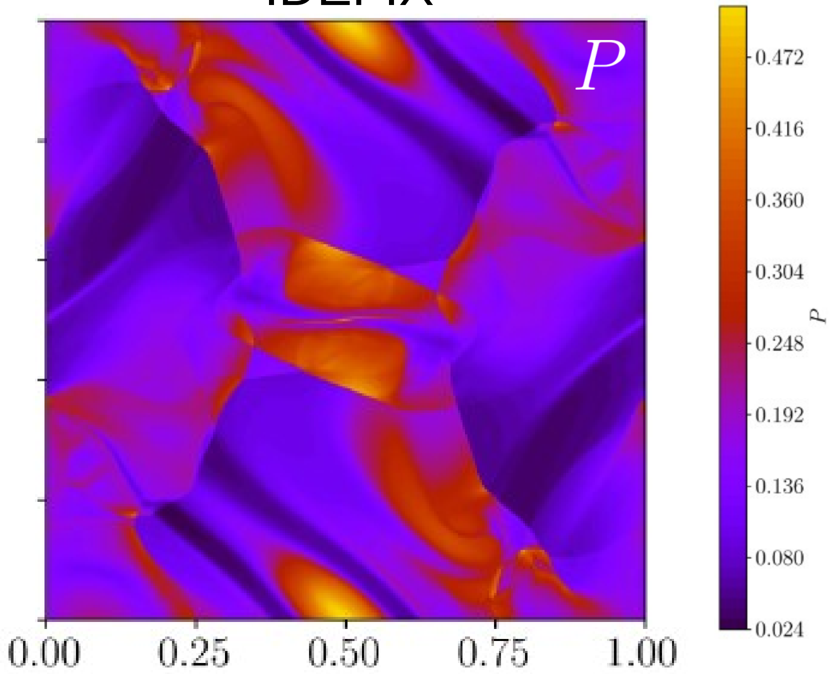
Functional Test: Orszag-Tang vortex

[Orszag & Tang 1979]

PHARE



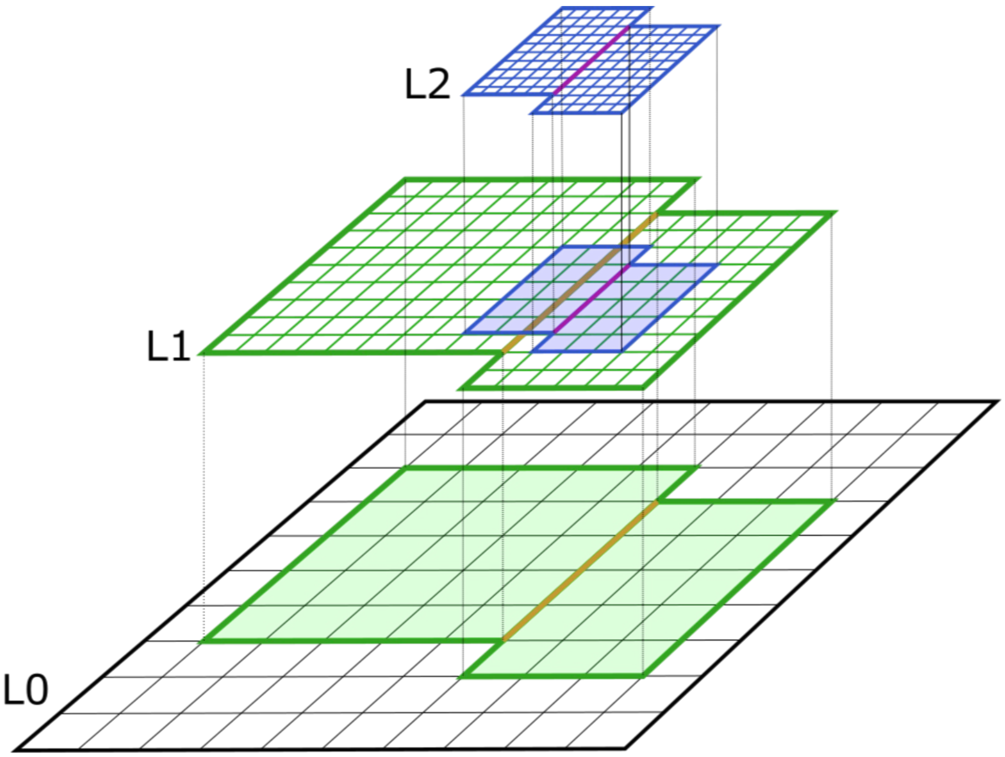
IDEFIX

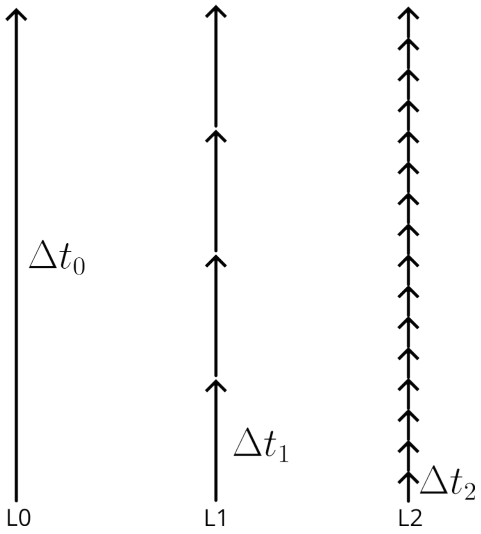


code IDEFIX: [Lesur et al. 2023]

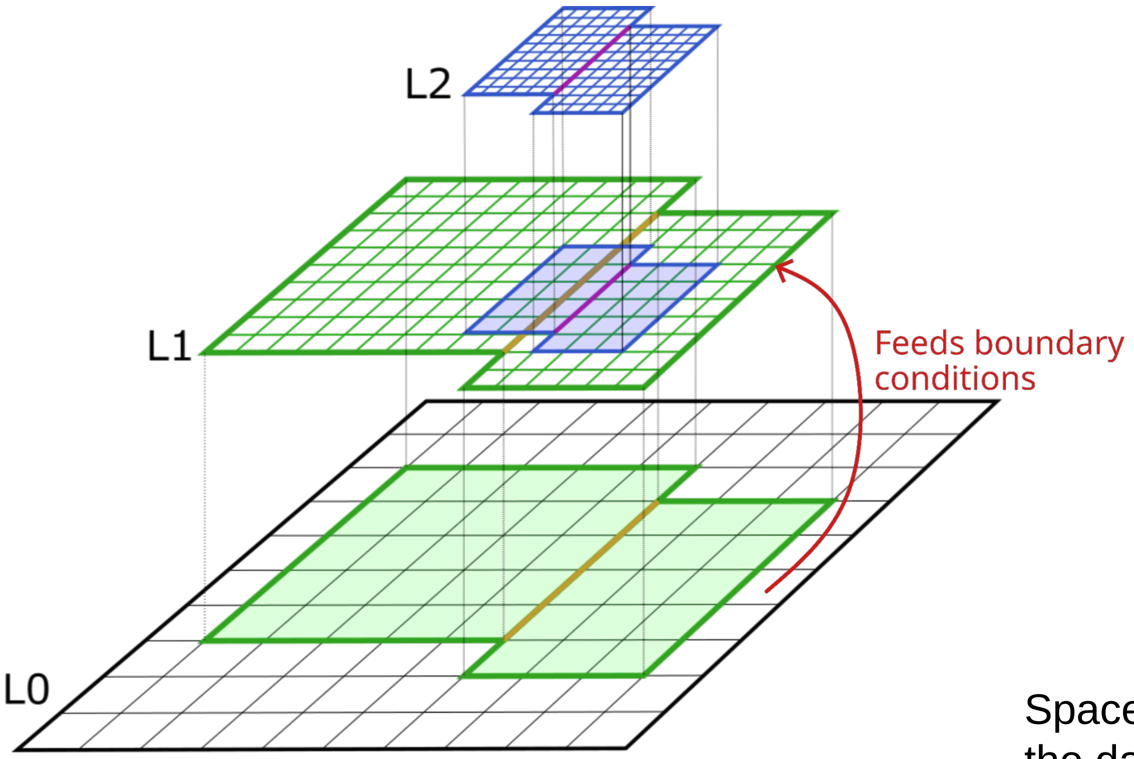
Pressure plot of the Orszag-Tang vortex at t=0.5 with a domain of 1, 1024x1024 cells and a second order scheme. PHARE (left) Idefix (right).

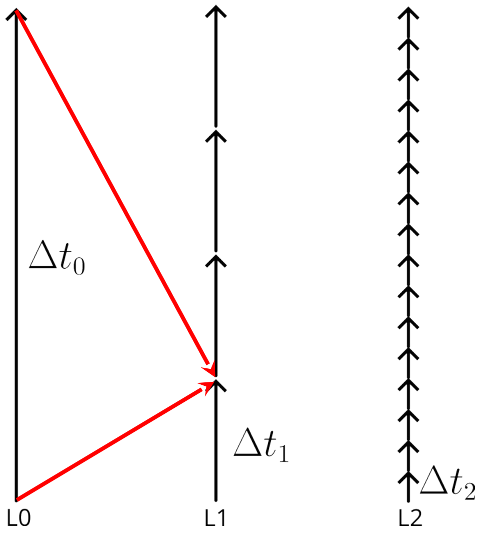
Step 3: Adaptive Mesh Refinement





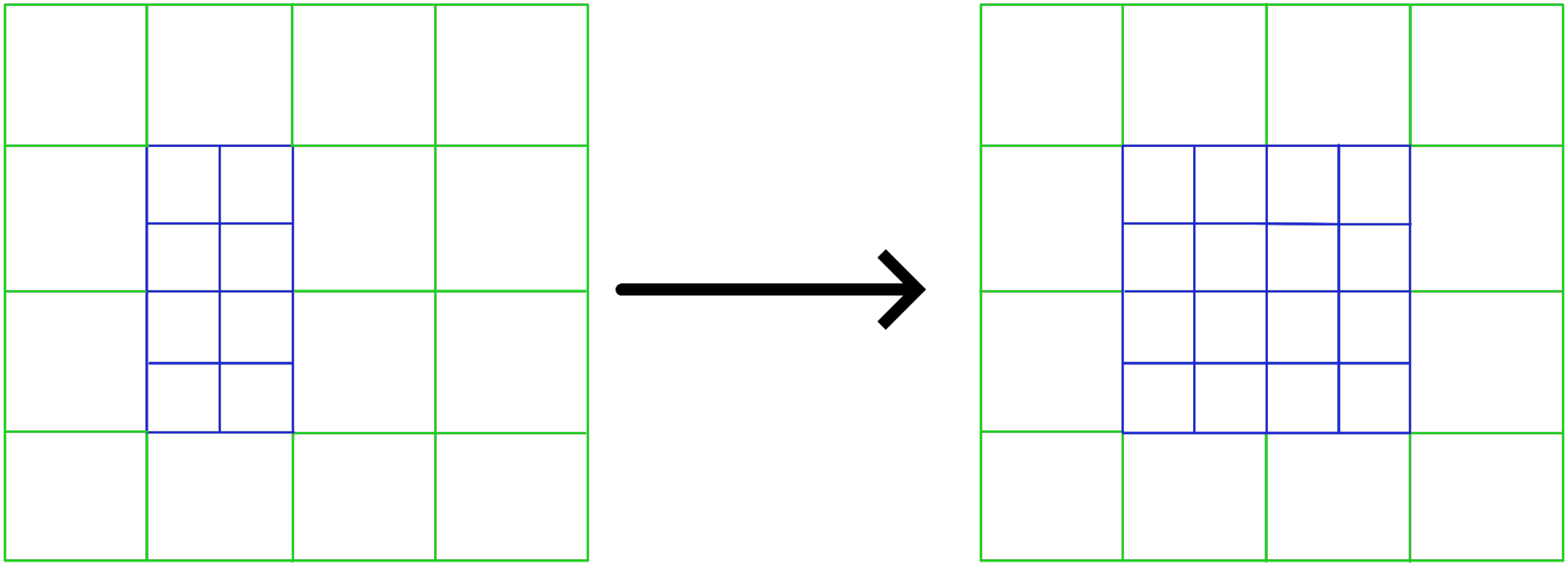
Step 3: Adaptive Mesh Refinement





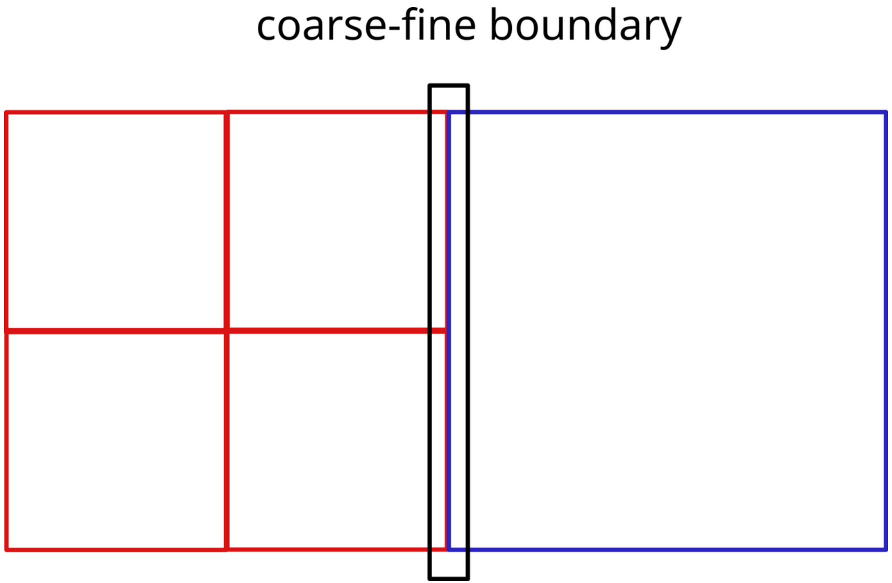
Space and time interpolation of the data at boundaries.

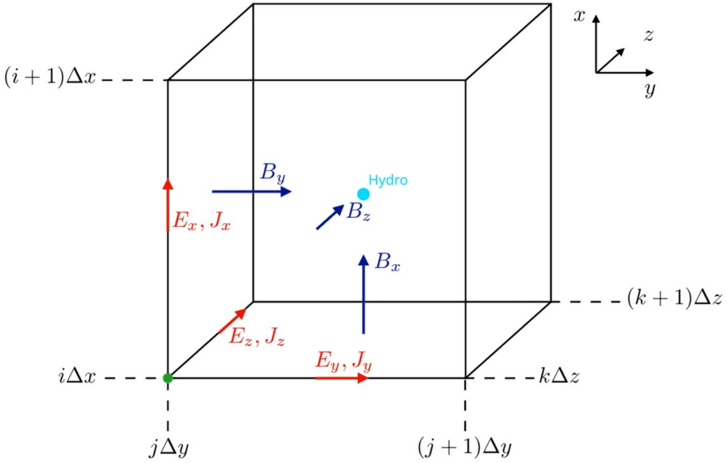
Step 3: Adaptive Mesh Refinement



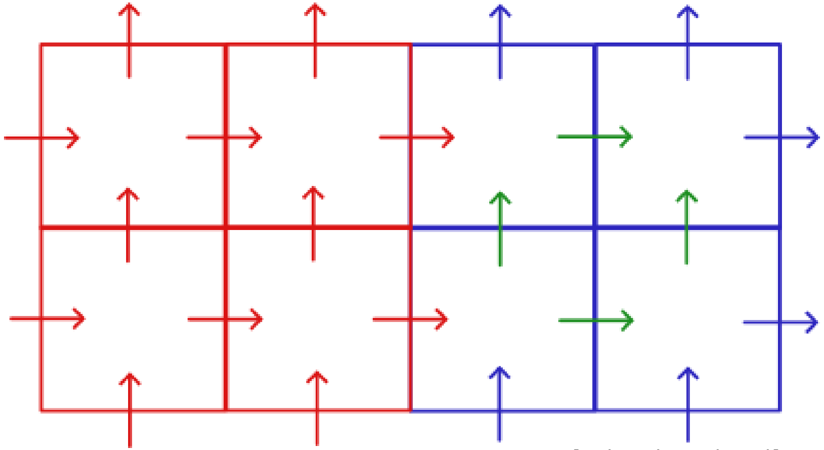
Adaptive mesh: grid can change to follow a criteria on the solution

The conservative refinement problem





The conservative refinement problem



[Toth and Roe (2002)]

$$u = B_x, v = B_y, w = B_z$$

$$u^{0,j,k} = \tfrac{1}{2}(u^{+,j,k} + u^{-,j,k}) + U_{xx} + k(\Delta z)^2 V_{xyz} + j(\Delta y)^2 W_{xyz}$$

$$v^{i,0,k} = \tfrac{1}{2}(v^{i,+,k} + v^{i,-,k}) + V_{yy} + i(\Delta x)^2 W_{xyz} + k(\Delta z)^2 U_{xyz}$$

$$w^{i,j,0} = \tfrac{1}{2}(v^{i,j,+} + v^{i,j,-}) + W_{zz} + j(\Delta y)^2 U_{xyz} + i(\Delta x)^2 V_{xyz}$$

with:

$$U_{xx} = \tfrac{1}{8}\textstyle\sum_{ijk=\pm} ijv^{i,j,k} + ikw^{i,j,k}$$

$$V_{yy} = \tfrac{1}{8}\textstyle\sum_{ijk=\pm} iju^{i,j,k} + jkw^{i,j,k}$$

$$W_{zz} = \tfrac{1}{8}\textstyle\sum_{ijk=\pm} iku^{i,j,k} + jkv^{i,j,k}$$

and:

$$U_{xyz} = \tfrac{1}{8}\textstyle\sum_{ijk=\pm} \frac{ijku^{i,j,k}}{(\Delta y)^2+(\Delta z)^2}$$

$$V_{xyz} = \tfrac{1}{8}\textstyle\sum_{ijk=\pm} \frac{ijkv^{i,j,k}}{(\Delta x)^2+(\Delta z)^2}$$

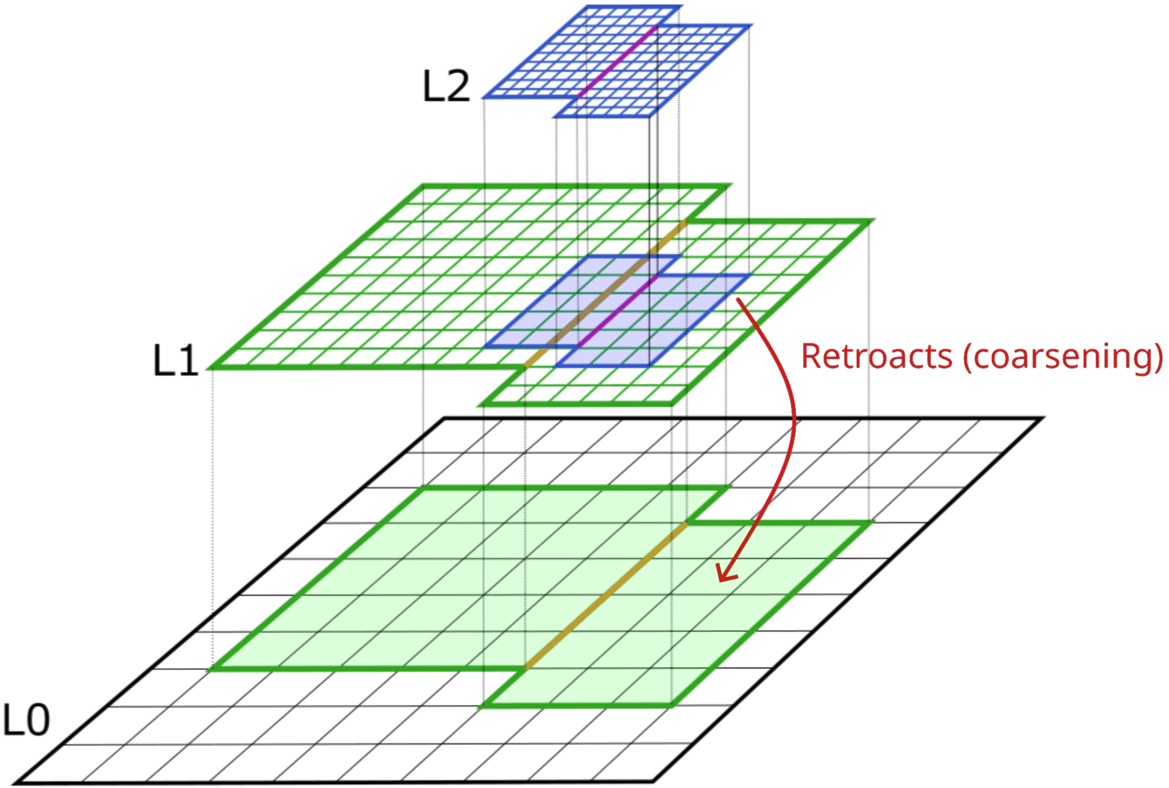
$$W_{xyz} = \tfrac{1}{8}\textstyle\sum_{ijk=\pm} \frac{ijkw^{i,j,k}}{(\Delta x)^2+(\Delta y)^2}$$

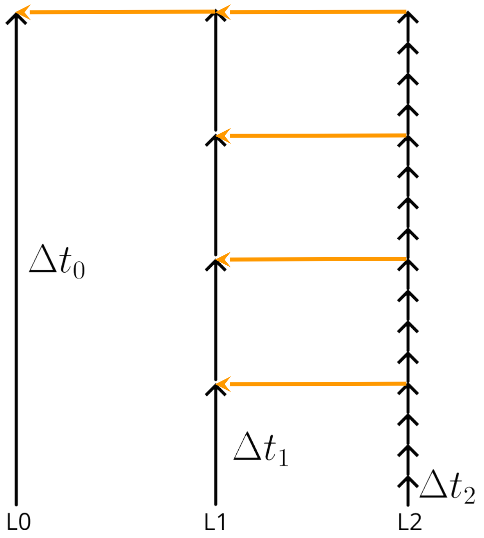
Idea: refine the new fine faces in such a way that:

$$\nabla \cdot \mathbf{B}_{fine} = \frac{1}{r^d} \nabla \cdot \mathbf{B}_{coarse}$$

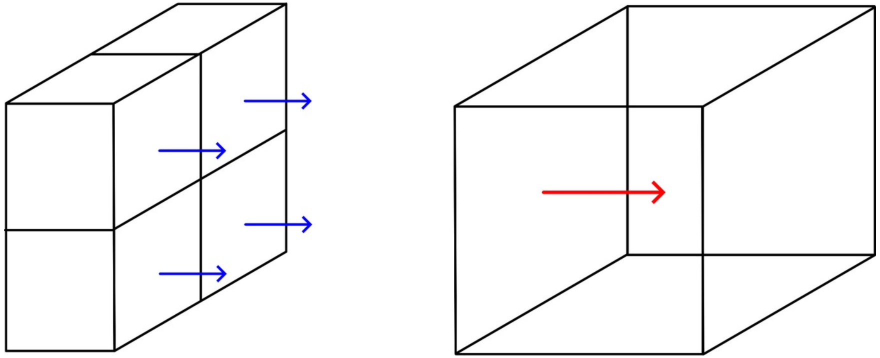
with r the refinement ratio and d the dimension.

Step 3: Adaptive Mesh Refinement



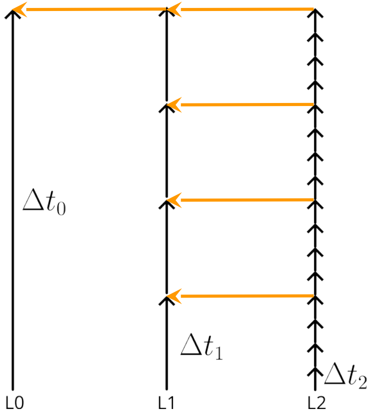


Conserving flux at coarse fine interfaces



Need to express the whole time integration step with a single flux. With multistep integrators, use Butcher flux:

$$\mathbf{U}^{n+1} = \mathbf{U}^{n} + \sum_{step} \gamma(\omega_{step}, \omega_{step-1}, \ldots, \omega_0)\mathbf{F}^{step}$$



$$\mathbf{F}^{total}_{fine} = \frac{1}{\mathcal{C}_{\text{sub-steps}} \times \mathcal{C}_{\text{shared fine faces}}} \sum_{\text{sub-steps}} \sum_{\text{shared fine faces}} \mathbf{F}_{fine}$$

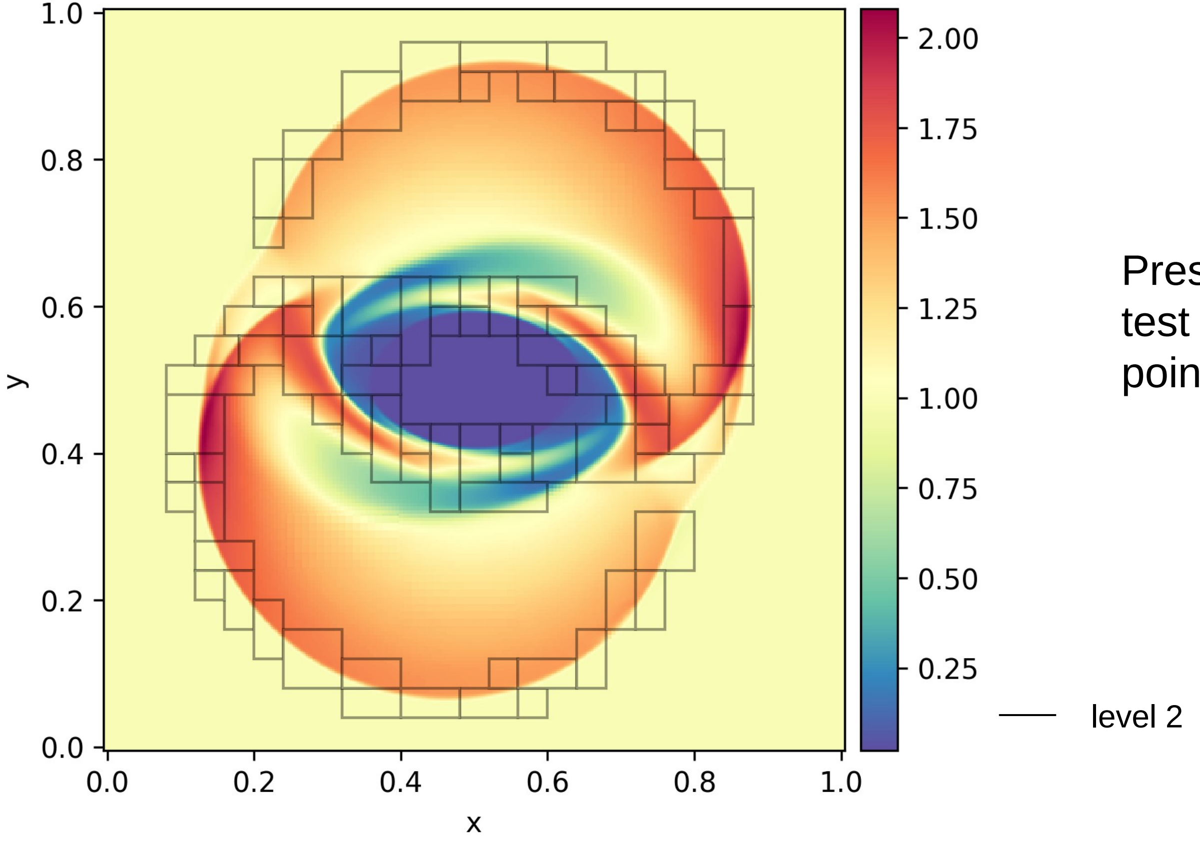
← Space-time average of the fluxes

$$\mathbf{U}^{n+1} = \mathbf{U}^{n+1} - \frac{\Delta t_{coarse}}{\Delta l} \left(\mathbf{F}_{coarse} - \mathbf{F}^{total}_{fine} \right)$$

← Correction step

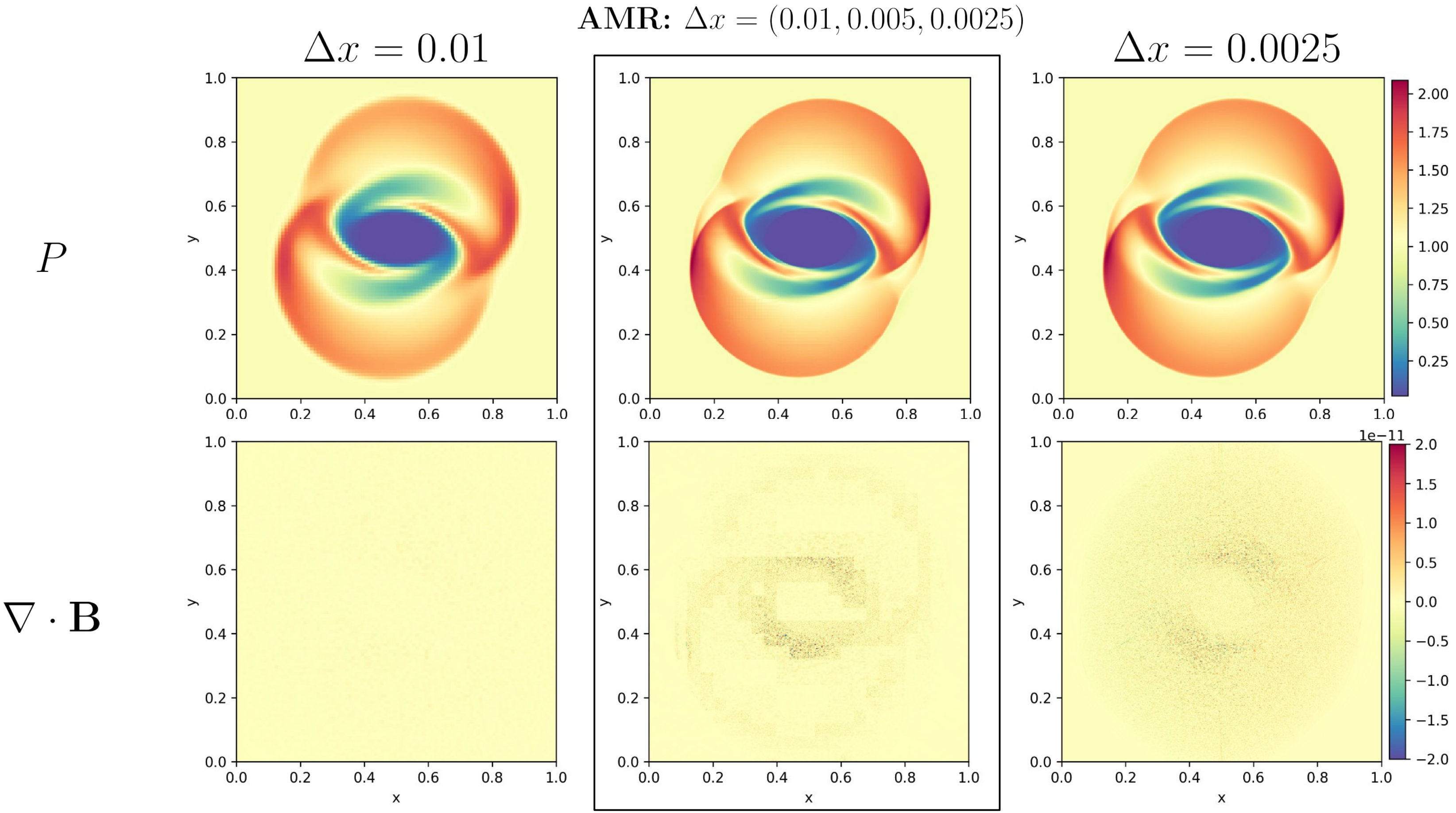
AMR Validation: MHD Rotor test

[Balsara & Spicer 1999]



Pressure for the MHD rotor test at t=0.15. 100x100 grid points, 3 AMR level.

AMR Validation: MHD Rotor test



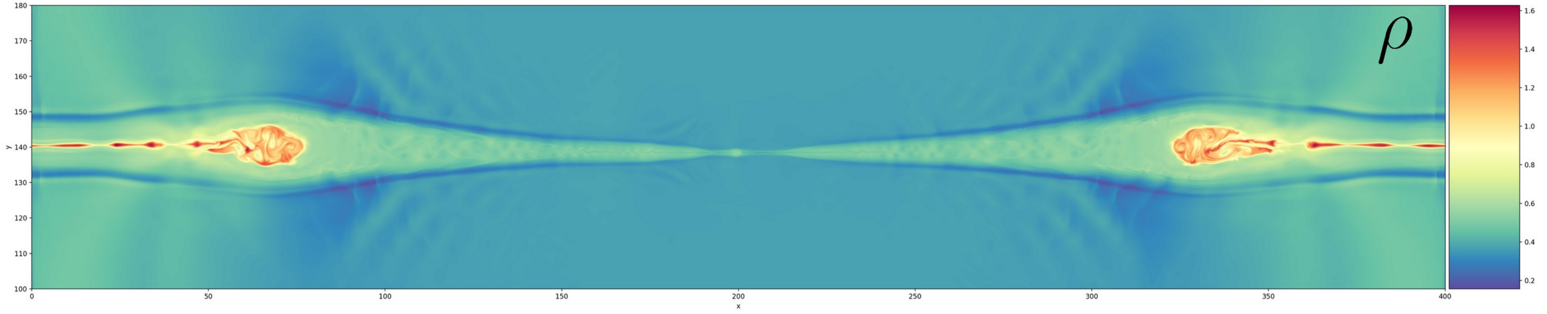
1000×500

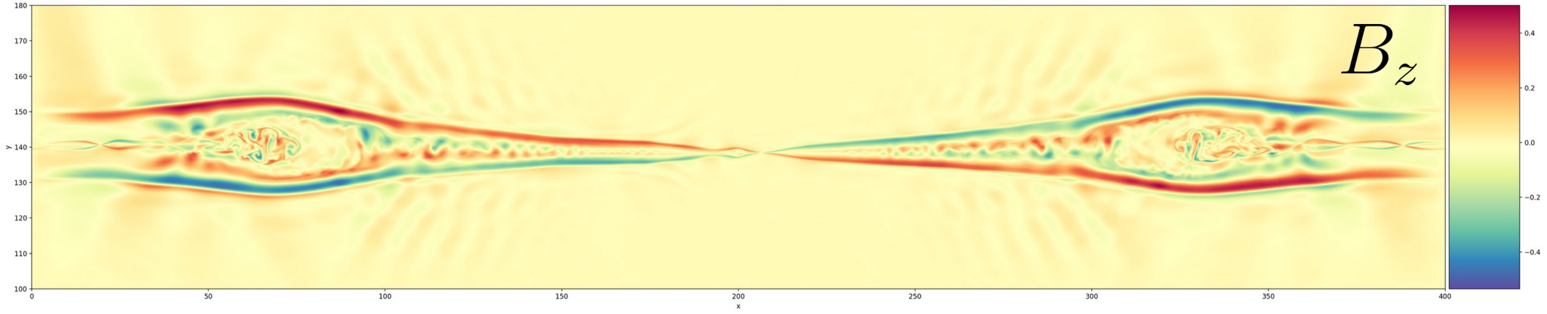
$\Delta x = 0.4$

3 AMR levels

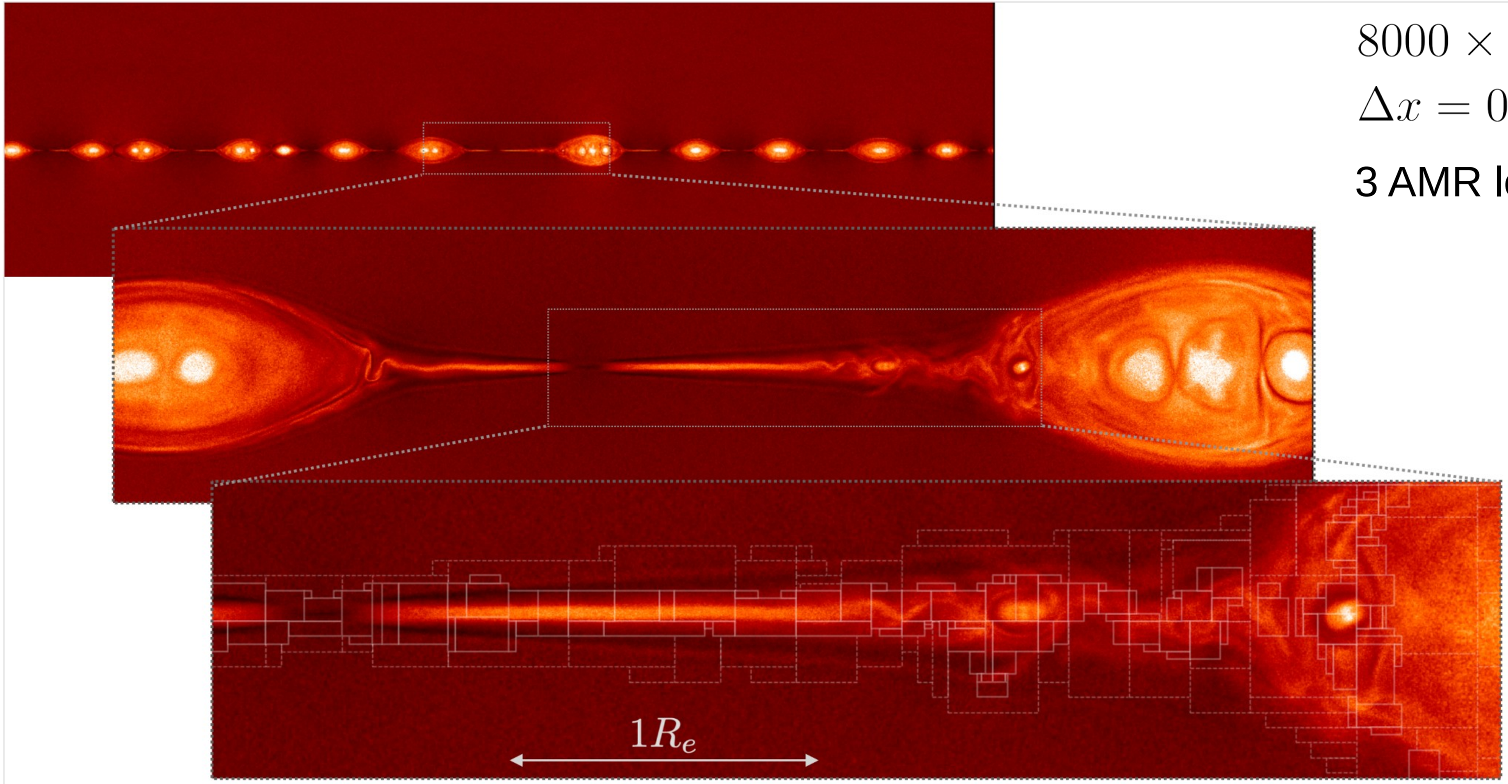
Symmetric current sheet

[Harris 1962]





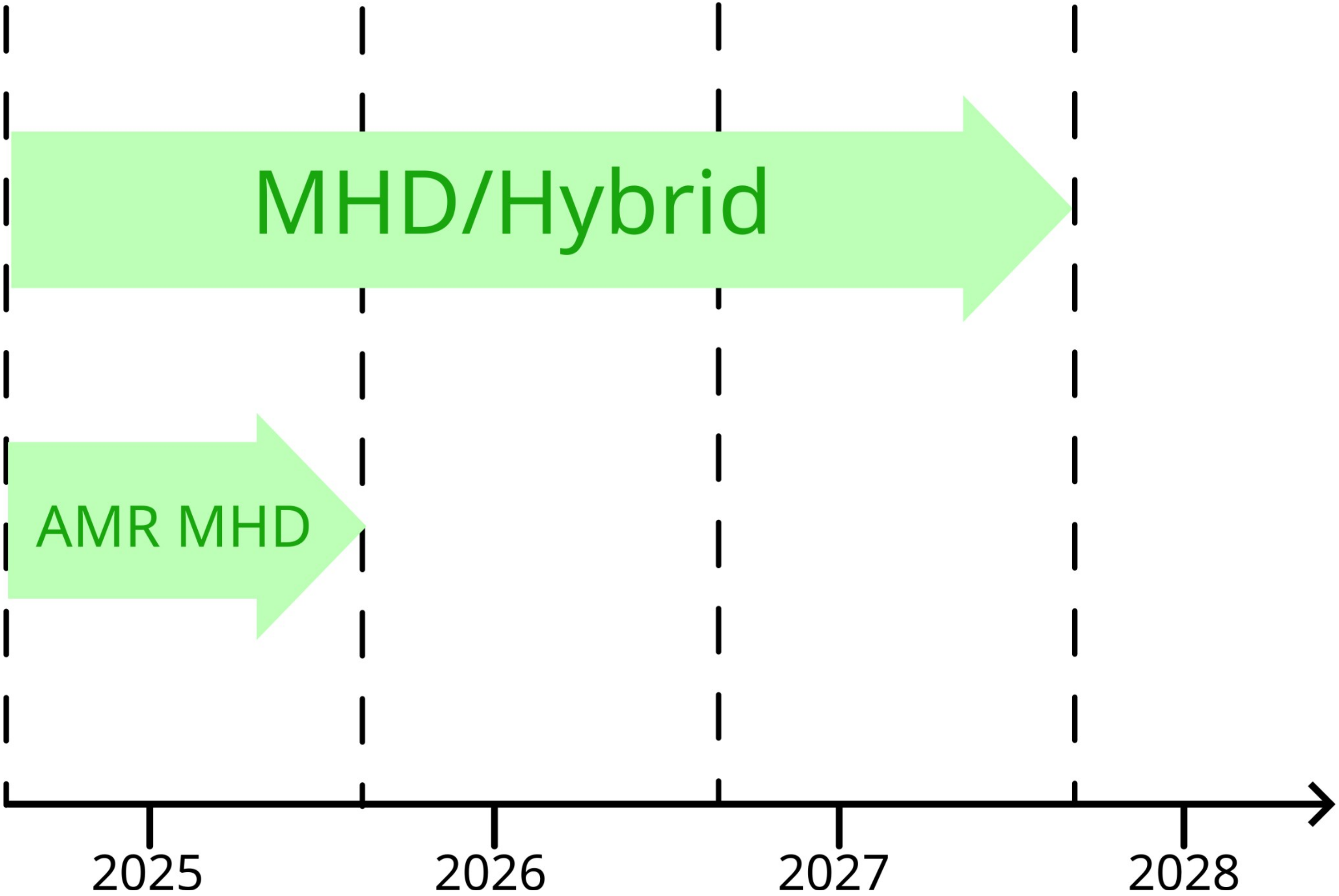
Symmetric current sheet with the hybrid solver

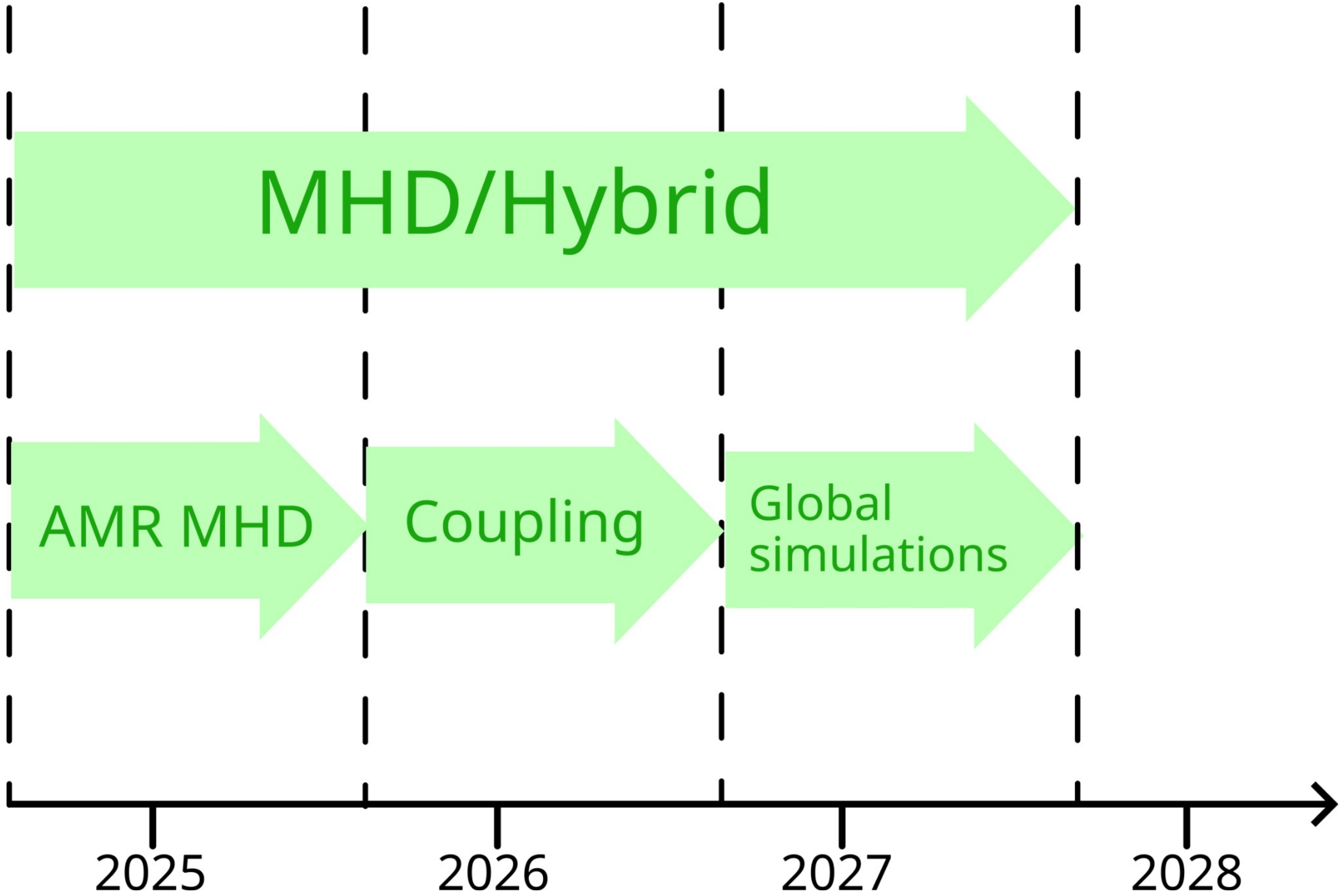


8000×4000

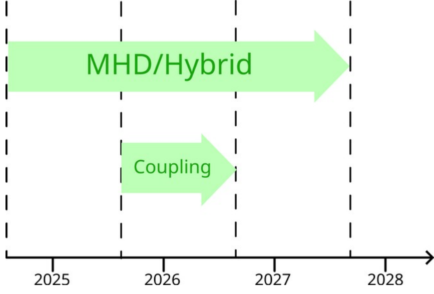
$\Delta x = 0.4$

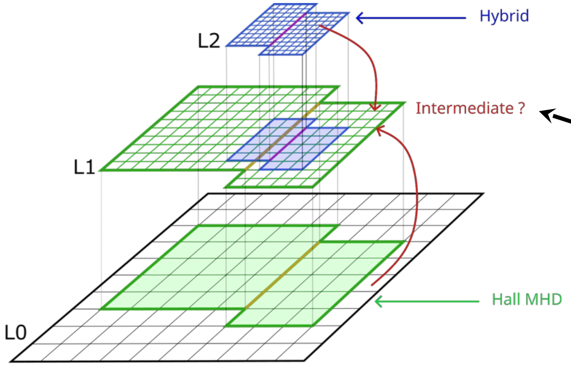
3 AMR levels


MHD/Hybrid
AMR MHD
2025
2026
2027
2028

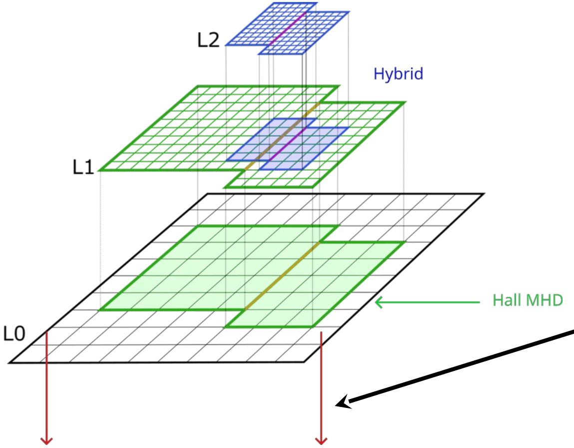

MHD/Hybrid
AMR MHD
Coupling
Global
simulations
2025
2026
2027
2028

Lots to experiment with





- Higher moment schemes ?



- Test particles in MHD ?
- Guiding center schemes ?

- Ideal MHD ?
- With asymptotic preserving schemes ?

Many new things coming to the code in the coming year

- Magnetospheric boundary conditions
- 3d visualisation
- Improved performances
- MHD/Hybrid coupling

→The main ingredients for our global simulations !